INTRAHOUSEHOLD INEQUALITY AND THE JOINT TAXATION OF HOUSEHOLD EARNINGS*

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Abstract

We derive optimal labor income tax schedules for married agents, taking the distinction between interpersonal and interhousehold inequality seriously. Each household consists of two workers with different productivity levels and unequal access to the family's economic resources. We handle the multidimensionality that could undermine the Mirrlees' (1971) approach by restricting preferences to be identical and iso-elastic and by focusing on taxes characterized by income-splitting. Individual-oriented utilitarianism typically leads to a misalignment between the households' and the government's objectives, which Apps and Rees (1988) have named *dissonance*. We show that optimal tax formulae are augmented by a Pigouvian term to account for dissonance. This term affects the direction of marginal tax rates, according to the correlation between the intra-household earnings gap and bargaining power. We also investigate the role of gender-based policies and find that when the least productive spouse is also the one with less power, the optimal policy will typically introduce a small tax on this agent.

Keywords: *Intrahousehold Inequality; Joint Taxation; Collective Household*
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RESOURCES are not equally split across household members (Calvi (2020); Anderson and Ray (2010)). This fact has significant and sometimes dramatic consequences for interpersonal and welfare inequality (Chavas et al. (2018); Chiappori and Meghir (2015); Haddad and Kanbur (1990)). Yet, most of the optimal tax theory does not directly face this important issue. To derive an optimal schedule for multi-person households, one must first deal with the multidimensional nature of the screening problem to which it gives rise. This poses well-known technical difficulties in a screening context – Rochet and Stole (2003). Indeed, advances in household tax theory have always been slowed by the need to confront these issues. As a consequence, most contributions to the literature limit its scope by pre-defining functional forms for taxes (Boskin and Sheshinski (1983)), precluding adjustments in all of the relevant decision margins (Kleven et al. (2009)), and/or restricting the amount of heterogeneity (Cremer et al. (2012)).

In the collective approach tradition, household welfare consists of a bargain-weighted average of each spouse's utility. The associated weights can differ from the planner's individual-oriented welfare metric. This raises the question: How does the potential misalignment between household and social objectives – dissonance, in the terminology of Apps and Rees (1988) – affect the design of optimal couple's taxation? We address this question by focusing on how the tax system can redistribute utility between spouses, even when the internal allocation of resources is not observed by the planner. We derive a Pigouvian adjustment of the traditional Mirrlees (1971) formula that accounts for dissonance. The Pigouvian term takes into account how changes in earnings affect the distribution of utility between spouses and how the direction of this transfer interacts with the distribution of power. With this, our formula sheds light on interpersonal rather than inter-household inequality.

Households are composed of two spouses, each with a utility function and an individual level of labor market productivity. Following the collective approach of household behavior — Chiappori (1988b, 1992); Apps and Rees (1988) — resources are allocated across spouses according to their relative power, captured by household-specific Pareto weights. Since the primitive object of concern for economic policies is the well-being of individuals, tax policy is assessed according to its consequence for each person, instead of the household.[1] While the government cares about individuals it does not directly observe how resources are shared

[1]By neglecting the multiperson nature of households, the welfare evaluation typical of optimal taxation literature is devoid of any meaning if we are committed to methodological individualism, a central tenet in economic analysis. Indeed, when coining the term "methodological individualism", Schumpeter (1954) wrote: "the self-governing individual constitutes the ultimate unit of social sciences; and that all social phenomena resolve themselves into decisions and actions of individuals that need not or cannot be further analyzed in terms of superindividual factors."

between spouses to directly influence the distribution of consumption within the household. What we show is that when designing the optimal tax schedule the planner must take into account how *utility* is transferred from one spouse to the other when distorting earnings choices.

To derive an optimal schedule for multi-person households, one must first deal with the multidimensional nature of the screening problem to which it gives rise. Toward this end, we assume that agents have identical iso-elastic preferences. In this case, by restricting the analysis to the case of income-splitting schedules, one can maintain full generality on skill distributions, the relative Pareto weight of spouses, and the correlation between spouses' skills while still solving the model. As in Heathcote and Tsujiyama (2021), the inherent multidimensionality that plagues optimal household taxation endogenously collapses into a single-dimensional problem in which the single-crossing property still holds. When compared to Heathcote and Tsujiyama (2021) the novel element in our analysis is that households assess their own well-being using a Bergsonian household utility function involving different Pareto weights applied to each spouse's utility whereas the planner uses individual-oriented utilitarianism, aggregating spouses' well-being as the sum of their utilities. The potential misalignment between household and social objectives creates the need for a Pigouvian correction term which can be explained by the product of two terms: one determining how changes in earnings affect the distribution of utility between spouses, and another determining the direction of utilitarian gains.

An increase (a decrease) in household earnings always transfers utility from the agent who works more (less) to the agent who works less (more). How the ratio of the power of the two spouses compares with the ratio of their productivity determines which spouse works more. For the case in which the least productive agent is also the one with less power – the more empirically relevant case – if the ratio of productivity is greater (lower) than the ratio of Pareto weights, then the planner will be optimal to discourage (encourage) work through higher marginal tax rates. When it is the least productive agent that has more power, then the analysis is inverted, the planner will try to transfer utility toward the more productive agent, which is accomplished through higher (lower) marginal tax rates when the ratio of productivity is greater (lower) than the ratio of Pareto weights. In our numeric exercise, we find that the Pigouvian term is always negative, leading to a more progressive system than the one that obtains if one disregard dissonance.

Under the maintained assumption of identical iso-elastic preferences, there is only one screening variable (household earnings), and three dimensions of characteristics (the two spouses' productivities and the relative weight parameter). Even though there is a bunching of different household types at each earning level, these sets of agents are identified by a common exogenous parameter. Importantly, the exogeneity of this aggregated parameter with respect to policy allows for an exogenous ordering of households' willingness to make effort. For general preferences, however, the ordering becomes endogenous to the tax system and the screening problem is not tractable. To deal with the general case we settle for a local characterization via perturbation methods which we present as an extension in Section 6.

We derive optimal tax formulae in terms of sufficient statistics with no restrictions on preferences, as in Saez (2001). The presence of dissonance introduces novel complexities for the aggregation of preferences at each income level as in Gerritsen's (2016) work on the taxation of behavioral agents. Without any assumptions on the degree of heterogeneity at each income level, new and hard-to-estimate statistics are needed to characterize the optimum (see equation (D.8) in the online Appendix D). Our analysis focuses on the case of joint income-splitting tax schedules.[2] We also discuss the welfare gains of gender-based taxation policies. We show that a small subsidy on the earnings of the least productive spouse may redistribute utility away from her.

The logic behind this result is that a spouse-specific income subsidy is a tax on her leisure. Under the assumption that household weights are not affected by the new policy, the distribution of consumption is not altered and the relative utility of the subsidized spouse may be reduced. This effect must, of course, be balanced by the overall decrease in welfare as perceived by the household. Still, when the least productive spouse is also the one with less power – the case emphasized by Immervoll et al. (2011) – optimal policy will typically introduce a small tax on this agent.[3]

The rest of the paper is organized as follows. After this introduction, we offer first the heuristic argument for dissonance emergence in our model and a brief literature review. Section 2 describes the economy. The policy objective is carefully discussed in Section 3. In Section 4, we show how the assumption of identi-

[2]Despite a growing tendency toward individual taxes, joint tax schedules are still pervasive. The usual rationale for their use is based on horizontal equity principles: one should not treat couples with identical earnings differently. Although we do not necessarily side with this view, we acknowledge the compelling argument by Gordon and Kopczuk (2014), who show that, from an equity perspective, joint taxes are a better departure point for tax design than individual taxes. Starting with a schedule that depends only on total household earnings (or on individual earnings in the case of individual taxation), Gordon and Kopczuk (2014) estimates the benefits of conditioning taxes on other observables. They show that departing from separate taxes requires more conditioning and accomplishes less in terms of promoting equity.

[3]This finding is also in line with Cremer et al. (2016). It *does* depend on the assumption adopted in almost all of the household taxation literature (Kleven et al. (2009); Immervoll et al. (2011); Cremer et al. (2016)) that household weights are invariant to perturbations to the policy but it *need not* be overturned and is, in fact, more likely to be reinforced by the presence of household production.

cal iso-elastic preferences allows us to reduce the dimensionality of the screening program to actually solve the model in terms of primitives. Section D generalizes the optimal formulae from Section 4 using a tax perturbation approach. These formulae are based on sufficient statistics that are not too easy to recover and that furthermore are endogenous to the policy. Our approach in this section highlights the type of assumptions that are needed if the approach is to be put into practice. Section 5 contains the quantitative exercises and the welfare implications of our model. In Section 6, we explore a simple form of gender-dependent perturbation on the joint schedule.

Heuristics

To provide intuition on how dissonance can affect the optimum, we start for didactic purposes with a simpler economy, which has only two types of households. In the example, the Mirrlees' solution uses the households' preferences as a normative criterion. The pair of earnings-consumption bundles, (z, x), that comprise it are depicted in Figure 1 at point A which is the intersection of purple indifference curves V_0 (for low productivity households), V_1 (for high productivity households) and at point B in V_1 which has the slope of the budget line, i.e., $45°$ line. The gray curves depict the planner's 'indifference curves'. When individual welfare is used as the normative criterion, the possibility of dissonance arises. In this case, how the planner ranks the different bundles may differ from how the household ranks them.

Point A depicts the bundle of earnings, z, and consumption, x, assigned to the low productivity household in the Mirrlees's optimum, for which household preferences are used as the normative criterion by the planner. At this point, the high-productivity household is indifferent between its own bundle and the low-productivity households'. Moreover, the low-productivity household has an indifference curve with a slope lower than $45°$ with downward distorted earnings.

With dissonance at the Mirrlees's optimum, the household welfare according to the planner is represented by the gray indifference curves W_0 for the low-productivity household and W_1 for the high-productivity household. The area to the southwest of the low-productivity household's bundle above the W_0 and below the V_0 curve represents all the bundles that are deemed better to the low-productivity household by the planner, but not by the household. For this large area in the figure, they are also incentive compatible since they are below V_1. As for feasibility, all the bundles below the $45°$ line are also feasible. Hence, the dashed area between the W_0 indifference curve and the highest $45°$ line defines the set of bundles that the planner considers better for the low-productivity household than the standard Mirrlees's optimal bundle. The *Better than Mirrlees* bundle depicted along the same $45°$ line is also incentive compatible as made clear by the fact that the high-productivity household's bundle lies in the lower contour set bounded by $\hat{V}_0$. It is apparent that there are even better incentive-feasible allocations, according to the planner's objective. Moreover, the configuration of the indifference curves is particular both in that we could have assumed different directions of dissonance, as we shall explain, and in the fact that at the Mirrlees' solution there is a region of superior (according to the planner) incentive-feasible allocations.[4] In general, reforms from a Mirrlees' optimum are more involved, but, are almost always possible.

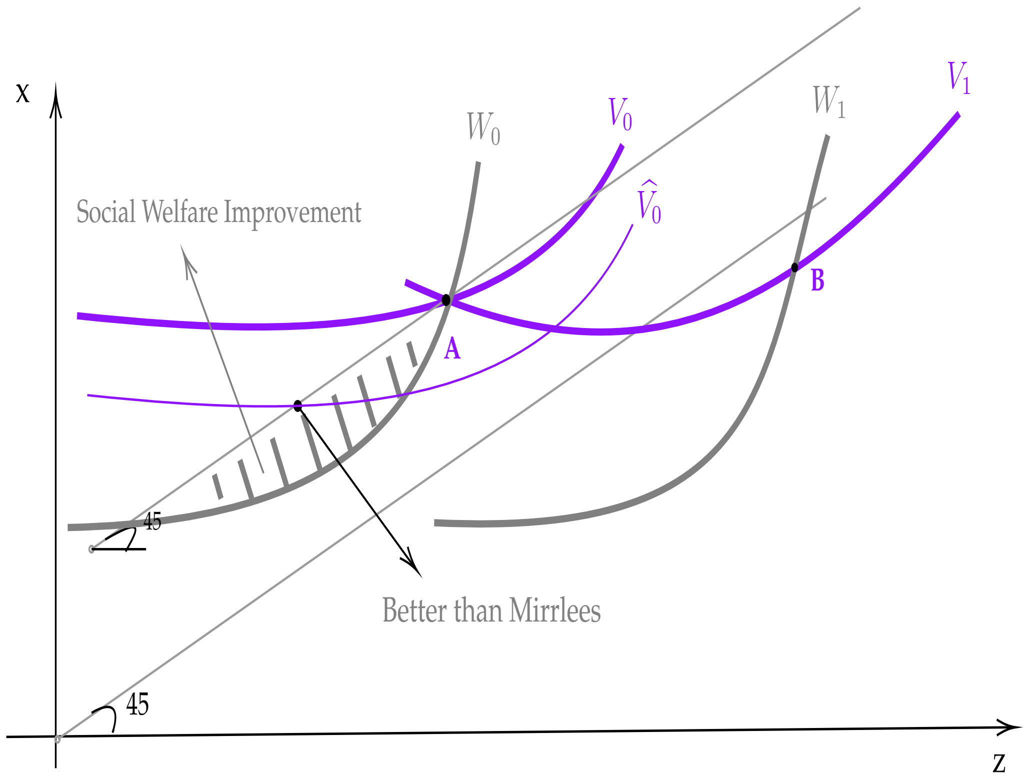

Figure 1: The standard Mirrlees's solution is the pair of bundles A, B. The gray indifference curves represent the planner's assessment of bundles for each household. Here, dissonance leads to steeper indifference curves for the planner, meaning that the planner would prefer households to work less than what they do in the standard Mirrlees's solution.

Related literature

Until recently, economists have approached household behavior on a common preference basis where all family members' resources were pooled to maximize a

[4]This configuration corresponds to the case in which the dissonance index, as defined in (6) is greater than one. The case in which it is less than one is equally plausible from a theoretical perspective.

single objective function. Moreover, this "household welfare function" had a normative status when used in optimal tax theory (e.g., Boskin and Sheshinski (1983)). However, starting with the pioneering contributions of Manser and Brown (1980) and McElroy and Horney (1981) and the conceptual change promoted by Chiappori (1988b, 1992); Apps and Rees (1988), Family Economics rapidly progressed toward the understanding of joint family decisions driven by divergent interests within the household.[5]

Boskin and Sheshinski (1983) explore the sub-optimality of tax schedules such as the one adopted in the US in which husbands and wives face equal marginal tax rates. They discuss the possibilities of taxing spouses equally or differently or even subsidizing one of the spouses. They show that taxing the earnings of husbands and wives at the same rate is inefficient because it disproportionately reduces female labor market participation. Boskin and Sheshinski (1983) use a unitary approach and take household (revealed) preferences as the normative criterion to be used by the planner. Moreover, they restrict their analysis to linear taxes. Bastani (2013) extends the analysis of linear taxes to a collective setting in which spouses decide through bargaining.

Also under a unitary approach, Kleven et al. (2009) analyze the general nonlinear optimal income tax for couples. In their model, the primary earner chooses labor supply as a continuum (intensive margin), while the secondary earner decides whether or not to participate in the labor market (extensive margin). If the secondary earner opts to participate in the job market, the labor supply is given, whereas, in our environment, both spouses choose in the intensive margin.[6] They show conditions under which the optimal tax scheme displays a positive tax on secondary earnings and when those taxes on secondary earnings decrease with primary earnings. Kleven et al.'s analysis starts from individual taxes and asks whether introducing jointness is optimal. Our analysis in Subsection 6.2 takes the exact opposite starting point (i.e., joint taxes) and asks whether introducing differentiation on marginal taxes can be welfare-improving.

Our assumption regarding household preferences and collective behavior implies the type of dimensionality-reduction exogenously imposed by Choné and Laroque (2010) while retaining the information that permits the assessment of individual welfare. A similar approach is used by Heathcote and Tsujiyama (2021) under the assumption of perfect insurance across large families. In their work, the distribution of power and dissonance are not considered.

[5]Lundberg and Pollak (1996), for example, adopted a bargaining protocol to model the independent agency of men and women in marriage. Early contributions to the understanding of the internal workings of families can be found in many of Gary Becker's contributions.

[6]In a previous version of our work, we explain how the analysis could be extended to allow for the secondary earner to adjust labor supply in both margins.

Closer to our work are the studies by Immervoll et al. (2011), Cremer et al. (2012), and Cremer et al. (2016). Dissonance, as defined by Apps and Rees (1988), plays a central role in all of these works. Immervoll et al. (2011) simplify their analysis by only considering extensive margin decisions and imposing strong restrictions on the choice sets. Cremer et al. (2012) handle multidimensionality by restricting couple types with the assumption of perfect assortative matching. Finally, Cremer et al. (2016) considers only a finite number of types. However, they consider fully general taxes and show how tax formulae change when compared to a world without dissonance. The small number of types they consider allows them to focus on a world in which incentive constraints only bind in the usual direction.

Guner et al. (2012) quantify the effects of tax reforms taking carefully into account the labor supply of married women as well as the current demographic structure. Married women have a heterogeneous cost of participating. They analyze how the structure of taxation can affect the participation decision, which is empirically responsive to tax perturbations. In our setting, both men and women have a heterogeneous participation cost, and they do not have the option to file separately if both spouses are working.

Our model is static, so we refrain from discussing the important issues related to the interaction between risk sharing and spouses' labor supply analyzed in Blundell et al. (2016) and Wu and Krueger (2021). However, as in these works, joint labor supply decisions are the essence of our analysis.

Finally, it is important to mention Gerritsen's (2016) study of behavioral agents. By distinguishing 'decision utilities' from 'experience utilities', a misalignment between individuals' and planners' objectives akin to dissonance arises in his work. Similar concerns with aggregation play a role in his optimal tax formulae. In our case, the presence of dissonance modifies the formula in two important ways. The first is the introduction of an additional sufficient statistic that should be taken into account: the wedge between the social values of income and its market price. The second concerns the aggregation of elasticities across households with the same earnings. The multi-dimensionality of household types leads to heterogeneity in preferences at each earning level.

This form of 'smooth bunching' is found in Saez (2001) since single-crossing is not imposed. The novelty here that arises from dissonance is that there are two different relevant aggregated elasticities: the usual average elasticities and a welfare-weighted elasticity.[7] Government revenues depend on average elasticities

[7]In Saez's words: "It is not necessary to assume that people earning the same income have the same elasticity; the relevant parameters are simply the average elasticities at given income levels" (Saez (2001), p. 210).

in which the average at each level of earning is calculated using the empirical distribution. Yet, elasticities are also important statistics because dissonance makes behavioral responses relevant to welfare.[8] In this case, the average elasticity is obtained by aggregating across households at each earning level using a welfare-adjusted distribution, analogously to the risk adjustment typically used in finance. This aggregation complexity also arises in Gerritsen's work. For the types of preferences typically used in optimal taxation studies, our optimal tax formula has an ABC plus D representation, which makes it a natural extension of the ABCs formulas first derived by Diamond (1998).

2 Environment

The economy is inhabited by a continuum of households (or families) with a measure normalized to one. Each household is composed of two spouses indexed by $i \in \{a, b\}$, where a identifies the female spouse and b, the male spouse in the household. Each spouse is characterized by a type $w_i \in [\underline{w}, \overline{w}] \subset \mathbb{R}_+$ denoting individual labor market productivity (or wage rate). Technology is linear as one efficient unit of labor produces one unit of consumption good. Agents are paid their marginal productivity, which implies that the labor income generated by spouse $i \in \{a, b\}$ with productivity w_i supplying l_i hours of work is given by $z_i = w_i l_i$. Individuals derive utility from the consumption of a private good $x_i \in \mathbb{R}_+$ and disutility from marketable labor supply $l_i \in \mathbb{R}_+$ resulting in a total utility given by $U(x_i, l_i)$, common to all individuals. Utility $U : \mathbb{R}_+^2 \to \mathbb{R}$ is assumed to be strictly quasi-concave, strictly increasing in consumption, strictly decreasing in labor supply, and of class $\mathcal{C}^2$.

Household decision making We take the multiperson nature of households seriously. We also assume that households are collective, in the sense that household consumption and labor supply choices always result in efficient outcomes, regardless of the bargaining protocol they engage in. Therefore, without loss of generality, we can assume that a household whose members have productivities (w_a, w_b) make consumption and labor supply decisions $(x_i, l_i)_{i \in \{a,b\}}$ to maximize a Bergsonian household utility function of the form

$$\alpha U(x_a, l_a) + (1 - \alpha) U(x_b, l_b), \tag{1}$$

[8] In particular, the envelope arguments used to restrict the welfare consequences of tax perturbations to the mechanical effect cannot be used if there is dissonance.

for some Pareto weight $\alpha \in [0,1]$, reflecting the relative contribution of the female spouse to household welfare. We impose no restrictions on the relationship between the Pareto weight, α, and spouses' productivities, (w_a, w_b). Given the previous structure, each household can be parameterized by a triple $(w_a, w_b, \alpha) \in \Lambda \equiv [\underline{w}, \overline{w}] \times [\underline{w}, \overline{w}] \times [0,1]$. This triple will henceforth be referred to as the family type, denoted by $\boldsymbol{\iota} \equiv (w_a, w_b, \alpha)$.

Informationally feasible allocations We follow the household economics literature by assuming that only total household consumption $x \equiv x_a + x_b$ is observable outside the household; that is, consumption cannot be assigned to a specific spouse. Because of this assumption, goods cannot be conditioned on each spouse's consumption.[9] Given this informational structure, bundles that can be associated with a household in any mechanism are of the form $(x, z_a, z_b) \in \mathbb{R}^3_+$.[10] Moreover, in most of this paper (except for Subsection 6.2), we restrict our analysis to policies that cannot distinguish the income generated by each of the spouses. In this case, feasible bundles are of the form $(x, z) \in \mathbb{R}^2_+$, where $z \equiv z_a + z_b$ is total income of the household. We call them *aggregate bundles*.

Tax system For most of what follows, we focus on joint taxation, which only uses information on total household income $z = z_a + z_b$, that is, $\mathbb{T}(z_a, z_b) = T(z_a + z_b)$.[11] Therefore, a government with a redistributive objective taxes households using a nonlinear joint income tax schedule, $T : \mathbb{R}_+ \to \mathbb{R}$, assigning a tax liability for each possible level of family income. Call $\mathcal{T}_0$ the set of all such tax systems. The net of tax income is used in the family consumption $x = z - T(z)$.

Studying joint taxation is particularly important because of its empirical relevance.[12] For instance, in the US, couples have the choice of filing taxes individually or jointly. Under the joint tax option, the marginal tax rates depend exclusively on the aggregate income of the household. The overall progressivity of the labor

[9] A more general approach is to define assignable and non-assignable goods. For the former, the spouse who is actually consuming each quantity can be determined, whereas, for the latter, this is not possible. One could also include public goods consumed by both spouses in a non-rivalrous manner. For simplicity, there is only one non-assignable private consumption good. This classification is independent of the relationship between taxation and labor choices emphasized in this paper.

[10] Equivalently, (x, z_a, z_b) is such an allocation if and only if $\phi(x, z_a, z_b) \leq 0$, for some function $\phi : \mathbb{R}^3_+ \to \mathbb{R}$. These are the only budget sets that can be designed to induce the desired allocation by a planner.

[11] The polar opposite is a separable tax schedule – a function $\mathbb{T} : \mathbb{R}^2_+ \to \mathbb{R}$ that assigns to each vector of household incomes $(z_a, z_b) \in \mathbb{R}^2_+$ a tax to be paid $\mathbb{T}(z_a, z_b) = T(z_a) + T(z_b) \in \mathbb{R}$, for some $\mathcal{C}^2$ function $T : \mathbb{R}_+ \to \mathbb{R}$. Any deviation from this separable tax system is not neutral to marital status and may impose a marriage penalty or bonus.

[12] Cremer et al. (2012) provides conditions under which the government should tax couples jointly.

income tax schedule means that filing individually is almost never optimal.[13] Although agnostic about the foundations of joint taxation, we view fairness as an appealing property for its use. This restriction on the space of feasible mechanisms will be convenient in reducing the number of margins that a government has to take into account in order to align incentives in this multidimensional environment.[14] Following Immervoll et al. (2011) we assume that α is exogenous and independent of policy.

Family problem As we have argued before, an important characteristic of the household consumption data is that most consumption is not assignable in the sense that only x, not x_a and x_b, is observed by outsiders. Moreover, under the restriction that policy instruments are based on total household earnings, the budget constraint is invariant to the source of labor income, and we can define for a type $\boldsymbol{\iota} = (w_a, w_b, \alpha)$ family a utility function $V : \mathbb{R}^2_+ \times \Lambda \to \mathbb{R}$ as

$$V(x, z, \boldsymbol{\iota}) \equiv \max_{(x_a,x_b),(z_a,z_b)\in\mathbb{R}^2_+} \left\{ \alpha U\left(x_a, \frac{z_a}{w_a}\right) + (1-\alpha)U\left(x_b, \frac{z_b}{w_b}\right) \quad \text{s.t.} \quad x_a + x_b = x \quad \text{and} \quad z_a + z_b = z \right\}. \tag{2}$$

The utility function V reflects the collective nature of family decisions and incorporates the intrahousehold allocation decisions of how much each spouse contributes to family income and how resources are distributed between family members.[15] It represents the ordering of (x, z) bundles by a household composed of spouses with productivities w_a and w_b and Pareto weight α. From this formulation, it is easy to see that different spouses from families choosing the same (x, z) can achieve very different utility levels depending on their relative productivities and Pareto weights.

[13] Ireland and Germany are some other countries that use this type of joint taxation.

[14] Mirrlees (1971) addresses the problem of choosing a full nonlinear tax system as a screening problem. It has long been recognized that screening problems in multidimensional environments are hard to handle because the absence of an exogenous ordering of willingness to pay or work precludes a simple characterization of the binding constraints. It is also well known that the optimum of these problems usually implies too much *pooling*; see Rochet and Choné (1998) and Rochet and Stole (1987).

[15] If we do not restrict the instruments beyond the informational constraint imposed by the non-assignability of consumption, the household utility for a type $\boldsymbol{\iota}$ family would be

$$\mathcal{V}(x, z_a, z_b, \boldsymbol{\iota}) \equiv \max_{(x_a,x_b)\in\mathbb{R}^2_+} \left\{ \alpha U(x_a, z_a/w_a) + (1-\alpha)U(x_b, z_b/w_b) \quad \text{s.t.} \quad x = x_a + x_b \right\}.$$

This last definition will be useful when we consider gender-based policies in Subsection 6.2. Of course, we can always define V in (2) through $V(x, z, \boldsymbol{\iota}) \equiv \max_{z_b\in\mathbb{R}_+} \mathcal{V}(x, z - z_b, z_b, \boldsymbol{\iota})$.

Two important assumptions are embedded in this formulation: α is exogenous to any policy, and the decision process within the family will always lead to an efficient outcome. This latter assumption places our approach in the realm of collective models (e.g., Chiappori (1988a) and Browning and Chiappori (1998)). As for the former, we could hold Pareto weights fixed for two plausible reasons. First, Pareto weights could be determined at the marriage stage, as in Gayle and Shephard (2019).[16] Second, as in da Costa and de Lima (2019), the planner may have additional instruments to handle Pareto weights. In both cases, an interesting question we leave for future work is how policies influence the distribution of weights in the long run through marriage markets. Since intrahousehold transfers are not observed, the allocation of consumption between spouses cannot be directly affected by any policy. Hence, for any feasible policy, household preferences ordering can be represented by the family utility function V.

3 The Planner's Problem

A utilitarian planner with fully flexible tax instruments would eliminate any intrahousehold inequality by setting effort and consumption across spouses accordingly. We take on the more realistic case in which the planner has access to imperfect instruments to influence intrahousehold inequality. In fact, we start with joint tax systems $T \in \mathcal{T}_0$ that do not discriminate between the sources of income. This section presents an optimal tax calculation to answer the question of how a government can use redistribution between households to achieve redistribution within the family. Family characteristics $\boldsymbol{\iota}$ are heterogeneous in the population and assumed to be private information of the family. Denote $F(\boldsymbol{\iota})$ as their joint cumulative distribution over the support Λ. The planner can use the tax policy $T : \mathbb{R}_+ \to \mathbb{R}$ to influence couples' aggregate decisions and spouses' allocations indirectly. We assume that the planner is restricted to using joint tax schedules $T \in \mathcal{T}_0$.

3.1 The planner's objective

We have associated with each household $\boldsymbol{\iota}$ a utility function $V(x, z, \boldsymbol{\iota})$ representing its preferences ordering on aggregate bundles. Whether any welfare meaning should be attached to it is a different matter.

[16]We do not take into account the policy impact on household formation. That is, we assume that couples do not divorce in response to changes in policies, nor do we allow spouses to anticipate those changes at the moment of marriage. Alm and Whittington (1999) find that in the US, the impacts of the income tax on marriage decisions, even when statistically significant, are small.

As Chiappori and Meghir (2015) have pointed out, a welfare metric that does not take into account intrahousehold inequality is inconsistent with any welfare criteria based on individuals, as is the case for utilitarianism. Traditional principles of morals and ethics do not justify the planner taking into account households' instead of individuals' utilities for welfare evaluation. Moreover, intrahousehold inequality is central to the study of redistribution policies. The goal of this paper is exactly to derive the implications of this intrahousehold inequality to the optimal tax theory. Therefore, even though the planner takes "household preferences" as given, its objective may depend arbitrarily on the utilities of all individuals. We shall focus on an anonymous individual-oriented utilitarian objective (i.e., the planner maximizes the average of all individuals' utilities).

Dissonance Under the individual-oriented utilitarian objective, the planner weights equally the utility of spouses in a given couple. Spouses, in contrast, decide using a different objective (1) captured by their Pareto weights α and $1-\alpha$. The solution to (2) defines consumption $x_a(x, z, \boldsymbol{\iota})$ and income $z_a(x, z, \boldsymbol{\iota})$ choices for the wife as a selection from the argmax correspondence. We can also directly define the analogous functions $x_b(x, z, \boldsymbol{\iota}) = x - x_a(x, z, \boldsymbol{\iota})$ and $z_b(x, z, \boldsymbol{\iota}) = z - z_a(x, z, \boldsymbol{\iota})$ for the husband. Using this notation, the household welfare *as perceived by the planner* is

$$W(x, z, \boldsymbol{\iota}) \equiv \frac{1}{2}\left\{V_a(x, z, \boldsymbol{\iota}) + V_b(x, z, \boldsymbol{\iota})\right\}, \tag{3}$$

where

$$V_i(x, z, \boldsymbol{\iota}) \equiv U\left(x_i(x, z, \boldsymbol{\iota}), \frac{z_i(x, z, \boldsymbol{\iota})}{w_i}\right), \quad i = a, b. \tag{4}$$

The function $W : \mathbb{R}_+^2 \times \Lambda \to \mathbb{R}$ represents the planner's assessment of individuals' welfare in a household $\boldsymbol{\iota}$ when consuming the aggregate bundle (x, z). Let z denote the household choice under a smooth tax schedule, $T \in \mathcal{T}_0$. We can in this case define the *dissonance index*, ξ, through the total derivative of W

$$\begin{aligned}\frac{dW}{dz} = \frac{\partial W(x, z, \boldsymbol{\iota})}{\partial x}[1 - T'(z)] + \frac{\partial W(x, z, \boldsymbol{\iota})}{\partial z} \\ = [1 - \xi(x, z, \boldsymbol{\iota})]\,\frac{\partial W(x, z, \boldsymbol{\iota})}{\partial x}[1 - T'(z)],\end{aligned} \tag{5}$$

where

$$\xi(x, z, \boldsymbol{\iota}) := \frac{\partial W(x, z, \boldsymbol{\iota})/\partial z}{\partial W(x, z, \boldsymbol{\iota})/\partial x}\left[\frac{\partial V(x, z, \boldsymbol{\iota})/\partial z}{\partial V(x, z, \boldsymbol{\iota})/\partial x}\right]^{-1} \tag{6}$$

is equal to one whenever $\alpha = 1/2$.

We can also re-write (3) as

$$W(x,z,\boldsymbol{\iota}) = V(x,z,\boldsymbol{\iota}) + \left(\frac{1}{2}-\alpha\right)\Bigg[U\left(x_a(x,z,\boldsymbol{\iota}), \frac{z_a(x,z,\boldsymbol{\iota})}{w_a}\right) - U\left(x_b(x,z,\boldsymbol{\iota}), \frac{z_b(x,z,\boldsymbol{\iota})}{w_b}\right)\Bigg]. \quad (7)$$

This expression has a nice behavioral analog.[17] The first term on the right-hand side of (7) is the spouses' aggregate utility under the household welfare metric (2). It is analogous to the *decision utility* of some behavioral economics models – see Farhi and Gabaix (2020). The planner's ordering of bundles, W, whose analog is the *experience utility* of behavioral economics does not coincide with the household's ordering (except when $\alpha = 1/2$). The difference between the two – the second term on the right-hand side of (7) – determines the misalignment between the household's and planner's welfare metric in the assessment of spouses' well-being which Apps and Rees (1988) called *dissonance*.

Note that, even if the ordering represented by $V(x,z,\boldsymbol{\iota})$ coincides with the one represented by $W(x,z,\boldsymbol{\iota})$ (leading to the same choices (x,z)), the value assigned by the planner to a unit of consumption good in the hand of each household may still depend on α. Indeed, how (x,z) is split between spouses matters and, in the case of equal preferences and productivity, unless household resources are equally divided between the spouses, that is, $x_a(x,z,\boldsymbol{\iota}) = x_b(x,z,\boldsymbol{\iota})$ and $z_a(x,z,\boldsymbol{\iota}) = z_b(x,z,\boldsymbol{\iota})$, there would be misalignment even if the household chose (x,z) exactly as the planner would.[18]

4 Optimal Taxation

By the revelation principle, the planner's problem can be restated in the space of direct mechanisms as

$$\max_{(x,z):\Lambda\to\mathbb{R}^2_+} \int_\Lambda W(x(\boldsymbol{\iota}), z(\boldsymbol{\iota}), \boldsymbol{\iota})dF(\boldsymbol{\iota}),$$

subject to incentive compatibility constraints: for every $\boldsymbol{\iota} \in \Lambda$

$$\boldsymbol{\iota} \in \arg\max_{\tilde{\boldsymbol{\iota}}\in\Lambda} V(x(\tilde{\boldsymbol{\iota}}), z(\tilde{\boldsymbol{\iota}}), \boldsymbol{\iota}),$$

[17] A couple in our model is analogous to a behavioral agent of Farhi and Gabaix (2020) and Gerritsen (2016).

[18] This is the case when instead of using equal weights to both spouses in (3), the planner would follow the weights α to the wife and $1-\alpha$ to the husband as in (1).

where we slightly abused notation in using $x(\cdot)$ and $z(\cdot)$ to denote the outcome function associated with the direct mechanism and the resource constraint:

$$\int_\Lambda [z(\boldsymbol{\iota}) - x(\boldsymbol{\iota})]\, dF(\boldsymbol{\iota}) \geq 0.$$

To circumvent the technical difficulties that arise from the multidimensional nature of the screening problem, we specialize our model in this section. We consider a class of preferences for which the household type collapses into a one-dimensional type with preferences that still possess the single-crossing property. Assume separable preferences of the form

$$U(x_i, l_i) = u(x_i) - h(l_i),$$

where $u(x_i) = \frac{x_i^{1-\sigma}-1}{1-\sigma}$ and $h(l_i) = \frac{l_i^{1+\gamma}}{1+\gamma}$, for $\sigma, \gamma > 0$. Notice that $u(x_i) = \ln x_i$ for $\sigma = 1$. Using $l_i = z_i/w_i$, we can re-parameterize each agent's utility as

$$U(x_i, z_i, w_i) = \frac{x_i^{1-\sigma} - 1}{1-\sigma} - \frac{1}{1+\gamma}\left(\frac{z_i}{w_i}\right)^{1+\gamma}.$$

These preferences are in line with the empirical evidence reviewed in Chetty (2006) and it is one of the most often parametrizations for preferences used in the taxation literature (see Tuomala (2016)). Under an income-splitting schedule, the earnings of a type $\boldsymbol{\iota} = (w_a, w_b, \alpha)$ household can be represented as if they were made by a representative individual with preferences

$$u(x) - \theta(\boldsymbol{\iota})h(z) \tag{8}$$

where

$$\theta(\boldsymbol{\iota}) := \frac{w(\boldsymbol{\iota})^{-1-\gamma}}{[\alpha^{1/\sigma} + (1-\alpha)^{1/\sigma}]^\sigma}$$

and

$$w(\boldsymbol{\iota}) := \left[\alpha^{\frac{-1}{\gamma}} w_a^{\frac{1+\gamma}{\gamma}} + (1-\alpha)^{\frac{-1}{\gamma}} w_b^{\frac{1+\gamma}{\gamma}}\right]^{\frac{\gamma}{1+\gamma}}$$

is the 'household productivity'.

To see this, consider a household's least costly way of producing z,

$$\mathscr{H}(z|\boldsymbol{\iota}) = \frac{1}{1+\gamma} \min_{z_a} \left\{ \alpha \left(\frac{z_a}{w_a}\right)^{1+\gamma} + (1-\alpha)\left(\frac{z - z_a}{w_b}\right)^{1+\gamma} \right\}.$$

It is easy to check that spouses' contribution to total household income can be expressed in terms of productivities, (w_a, w_b), and the Pareto coefficient α as

$$\frac{z_a}{z} = \underbrace{\alpha^{\frac{-1}{\gamma}} \frac{w_a^{\frac{1+\gamma}{\gamma}}}{w(\boldsymbol{\iota})^{\frac{1+\gamma}{\gamma}}}}_{k_a} \quad \text{and} \quad \frac{z_b}{z} = \underbrace{(1-\alpha)^{\frac{-1}{\gamma}} \frac{w_b^{\frac{1+\gamma}{\gamma}}}{w(\boldsymbol{\iota})^{\frac{1+\gamma}{\gamma}}}}_{k_b}.$$

Substituting z_a and z_b in (4) gives us

$$\mathscr{H}(z|\boldsymbol{\iota}) = \frac{1}{1+\gamma}\left(\frac{z}{w(\boldsymbol{\iota})}\right)^{1+\gamma}.$$

A spouse's contribution to household income varies negatively with the spouse's own individual Pareto weight; not only consumption but also leisure increases with one's Pareto weight. Individual earnings also vary positively with the spousal productivity gap, which means that more productive spouses work relatively more, holding all other primitives fixed. Now let us consider the allocation of consumption between spouses. A simple application of Borch's rule leads to

$$\mathscr{U}(x|\boldsymbol{\iota}) = \max_{x_a}\left\{\alpha\frac{x_a^{1-\sigma}-1}{1-\sigma} + (1-\alpha)\frac{(x-x_a)^{1-\sigma}-1}{1-\sigma}\right\} = \frac{B(\alpha)x^{1-\sigma}-1}{1-\sigma},$$

where $B(\alpha) = \left[\alpha^{1/\sigma} + (1-\alpha)^{1/\sigma}\right]^{\sigma}$. Combining both facts, we have:

$$V(x,z,\boldsymbol{\iota}) = \mathscr{U}(x|\boldsymbol{\iota}) - \mathscr{H}(z|\boldsymbol{\iota}) = B(\alpha)\left[\frac{x^{1-\sigma}-B(\alpha)^{-1}}{1-\sigma} - \theta(\boldsymbol{\iota})\frac{z^{1+\gamma}}{1+\gamma}\right],$$

for $\sigma \neq 1$ and

$$V(x,z,\boldsymbol{\iota}) = \mathscr{U}(x|\boldsymbol{\iota}) - \mathscr{H}(z|\boldsymbol{\iota}) = \kappa(\alpha) + \ln x - \theta(\boldsymbol{\iota})\frac{z^{1+\gamma}}{1+\gamma},$$

for $\sigma = 1$, where $\kappa(\alpha) \equiv \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha)$.

Two features are worth noting in the household utility function (8). First, family types only enter the objective through $\theta(\boldsymbol{\iota})$. Therefore, families whose characteristics generate the same $\theta(\boldsymbol{\iota})$ share the same objective. We can think of $\theta(\boldsymbol{\iota})$ as a sufficient statistic for the family characteristics when computing the value it assigns to a given bundle $(x,z) \in \mathbb{R}_{++} \times \mathbb{R}_{+}$. Second, marginal rates of substitution $\theta(\boldsymbol{\iota})h'(z)/u'(c)$ are exogenously ordered and we can follow Mirrlees' approach: screening households by their unitary type $\theta(\boldsymbol{\iota})$.

In other words, the combination of iso-elastic preferences for spouses and joint income-splitting taxes allows us to collapse the household's multi-dimensional type $\boldsymbol{\iota}$ into a uni-dimensional aggregator that preserves single-crossing. Assuming the existence of such an aggregator allows Choné and Laroque (2010) to study taxation in a multi-dimensional setting. Here we show that, as in Heathcote et al. (2017), household cooperative behavior implies the existence of such an aggregator.

In all that follows we focus on the case $\sigma = 1$. This will simplify many of our expressions and allow us to obtain closed-form expressions for some of the extensions we study. The counterpart for the dissonance expressions we derive is a trivial generalization of the ones we use here. The only feature that specializing for $\ln$ preferences eliminates is the possible dependence of marginal social welfare weights on α.[19] While interesting *per se*, the possibility that Pareto weights vary with aspects of an agent type has already been studied in other contexts, e.g., Choné and Laroque (2010), so we skip it for brevity.

The particular parametrization of preferences we choose implies that both spouses always supply a positive quantity of labor. Hence, decisions are taken along the intensive margin only. In a previous version of the paper – Alves et al. (2021) – we discuss how participation decisions can be introduced in an extension of the model where an additional discrete participation cost is introduced in the model. This would allow us to better approximate the empirical differences in labor supply elasticities of married men and women.

4.1 Optimal tax formulae

We consider a government that taxes households using a nonlinear joint income tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$ to maximize a utilitarian social welfare function. For every θ, it will be convenient to define the set $\mathcal{I}(\theta) \equiv \{\boldsymbol{\iota} \in \Lambda | \theta(\boldsymbol{\iota}) = \theta\}$ of all family types $\boldsymbol{\iota}$ whose behavior is summarized by the same θ. As we have seen, for any $\boldsymbol{\iota}$-household such that $\boldsymbol{\iota} \in \mathcal{I}(\theta)$, the problem optimization problem it faces when a tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$ is in place is

$$\max_{(x,z)\in\mathbb{R}_{++}\times\mathbb{R}_+} \left\{ \ln x - \theta \frac{z^{1+\gamma}}{1+\gamma}, \quad \text{s.t.} \quad x = z - T(z) \right\}. \tag{9}$$

Also, let us define the cumulative distribution of θ as

$$\Psi(\theta) = \int_{[\theta(\boldsymbol{\iota})\leq\theta]} dF(\boldsymbol{\iota}) \tag{10}$$

and assume that its associated density $\psi(\theta)$ is well defined and strictly positive. This allows us to think about the planner's program as if it were a program involv-

[19]The marginal social value of one unit of consumption in the hands of a couple with net income x and household weight α is $0.5\left[(\alpha)^{1/\sigma} + (1-\alpha)^{1^{\sigma}}\right]^{\sigma} x^{1-\sigma}/(1-\sigma)$ for $\sigma \neq 1$ and $\ln x$ for $\sigma = 1$.

ing a representative individual for the family whose type is the one-dimensional parameter θ and derives utility from the aggregate bundle (x, z). Aggregate 'household preferences' are well-behaved and exogenously ordered with respect to the parameter θ (i.e., the *Spence-Mirrlees condition* is satisfied). The first-order necessary condition for the type θ family's problem (9) is

$$1 - T'(z) = x\theta z^{\gamma}.$$

To write the planner's problem let us define

$$\bar{W}(x, z, \theta) \equiv \left[W(x, z, \boldsymbol{\iota}) \middle| \boldsymbol{\iota} \in \mathcal{I}(\theta)\right] \quad \text{and} \quad \bar{V}(x, z, \theta) \equiv \left[V(x, z, \boldsymbol{\iota}) \middle| \boldsymbol{\iota} \in \mathcal{I}(\theta)\right],$$

for all θ. Next, if we average $\xi(x, z, \boldsymbol{\iota})$ across all $\boldsymbol{\iota} \in \mathcal{I}(\theta)$ we define a 'θ-household average wedge',

$$\bar{\xi}(\theta) \equiv \mathbb{E}\left[\xi(x, z, \boldsymbol{\iota}) \middle| \boldsymbol{\iota} \in \mathcal{I}(\theta)\right], \tag{11}$$

which we will refer to simply as the θ-household wedge, and note that[20]

$$\bar{W}(x, z, \theta) = \bar{V}(x, z, \theta) + \left[1 - \bar{\xi}(\theta)\right] \theta \frac{z^{1+\gamma}}{1+\gamma}.$$

This allows us to write the planner's objective as

$$\max_{(x(\theta), z(\theta))_{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \bar{V}(x(\theta), z(\theta), \theta) + \left[1 - \bar{\xi}(\theta)\right] \theta \frac{z(\theta)^{1+\gamma}}{1+\gamma} \right\} \psi(\theta) d\theta, \tag{12}$$

while keeping the incentive compatibility constraint[21]

$$\theta \in \underset{\hat{\theta}}{\operatorname{argmax}}\, V(x(\hat{\theta}), z(\hat{\theta}), \theta),$$

and the resource constraint

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[z(\theta) - x(\theta)\right] \psi(\theta) d\theta \geq G,$$

identical to the ones in the single person Mirrlees' program. Solving the planner's program we arrive at the main proposition of this section.

Proposition 1. *In all intervals where $z(\theta)$ is strictly increasing, the marginal tax rate for*

[20] While $\xi(x, z, \boldsymbol{\iota})$ is allowed to depend on (x, z), under the specification of preferences we are using, it does not.

[21] Note that all households $\boldsymbol{\iota}$ such that $\boldsymbol{\iota} \in \mathcal{I}(\theta)$ rank (x, z) bundles the same way. From an incentive perspective the distinction between V and $\bar{V}$ is, therefore, inconsequential.

the optimal (nonlinear) income joint taxation is

$$\frac{T'\left(z(\theta)\right)}{1-T'\left(z(\theta)\right)}=\frac{1}{\psi(\theta)x(\theta)\theta}\int_{\underline{\theta}}^{\theta}\left[x(s)-\mathbb{E}^{\psi}[x]\right]\psi(s)ds-\frac{\mathbb{E}^{\psi}[x]}{x(\theta)}\left[1-\bar{\xi}(\theta)\right]. \quad (13)$$

Proof. See Appendix A. □

4.2 A Pigouvian Connection

According to Proposition 1, the Mirrlees' optimal tax formula must be adjusted to include a Pigouvian term that accounts for the differential impact of changes in the redistribution of utility across spouses. To understand this effect, take the definition of $V_i(x,z,\boldsymbol{\iota})$, for $i=a,b$, from (4). Then, we can write, at any z, the difference in utility attained by spouses as

$$V_a(z-T(z),z,\boldsymbol{\iota})-V_b(z-T(z),z,\boldsymbol{\iota})=\ln\frac{\alpha}{1-\alpha}-\left[\left(\frac{k_a}{w_a}\right)^{1+\gamma}-\left(\frac{k_b}{w_b}\right)^{1+\gamma}\right]\frac{z^{1+\gamma}}{1+\gamma}$$

where k_i, $i=a,b$, is the share of z earned by spouse i – see (4). Hence,

$$\frac{dV_a}{dz}-\frac{dV_b}{dz}=-\left[\left(\frac{k_a}{w_a}\right)^{1+\gamma}-\left(\frac{k_b}{w_b}\right)^{1+\gamma}\right]z^{\gamma}. \quad (14)$$

Noting that $zk_i/w_i=n_i$, $i=a,b$, it is apparent that a policy that induces a slight change in z affects differently the utility of the two spouses whenever $n_a\neq n_b$. In particular, if $n_a>n_b$, then increasing z will increase more (decrease less) the utility of spouse b than that of spouse a. The logic is easy to grasp. A marginal increase in z has the same impact on the utility of consumption of both spouses due to the $\ln$ utility assumption. As for the change in the disutility of effort, under the equal iso-elastic preferences assumption, the elasticity of z_i with respect to z is one; whoever works more, suffers more from an increase in z. The right-hand side of (14) shows that it is the ratio between relative productivities to relative power that determines which spouse works more.[22]

The right-hand side of (14) also offers a condition under which it is possible to change the relative utilities of the two spouses; whenever the ratio of relative power to relative productivities is not one, the sign of dz can be chosen to move relative utilities in whatever direction the planner desires. Of course, it may not be desirable to do so. A discussion to which we turn next.

[22] When $\alpha=1/2$, it is the most productive spouse that works more. In this case, increases in z benefit him (her) less than benefits his (her) spouse.

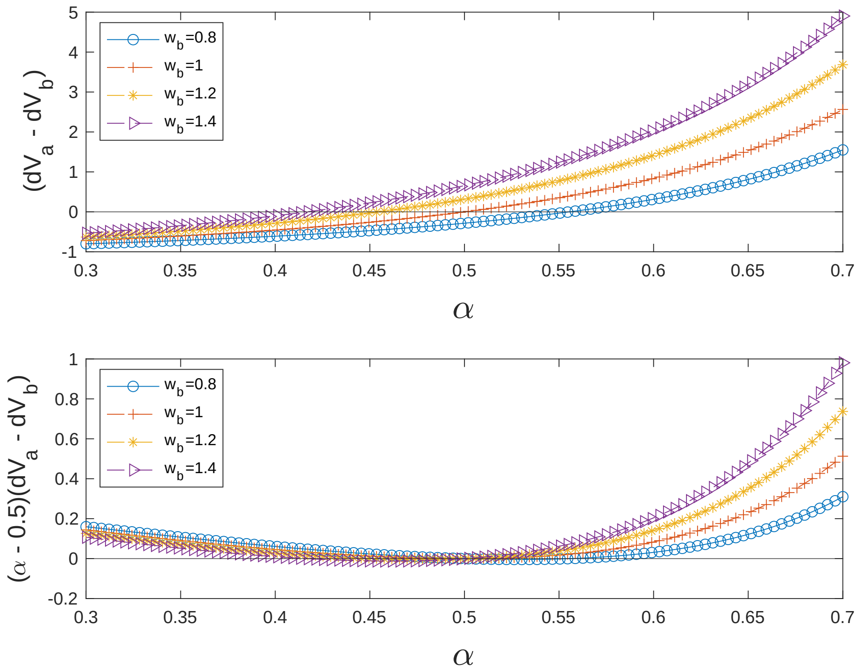

Figure 2: The figure in the top plots the utility difference between spouses as a function of the bargain weights α and the within-household income difference. Each line represents a different productivity ratio ($w_a = 1$), while the x-axis varies the bargain weights. The figure at the bottom displays the utilitarian gains from increasing earnings.

The Utilitarian Consequence From the Utilitarian planner's perspective the impact of a small change in z is

$$\frac{dV_a}{dz} + \frac{dV_b}{dz} = \frac{1 - T'(z)}{z - T(z)} - \frac{1}{2}\left[\left(\frac{k_a}{w_a}\right)^{1+\gamma} + \left(\frac{k_b}{w_b}\right)^{1+\gamma}\right] z^{\gamma},$$

which, using the household's optimality condition, can be written[23]

$$\begin{aligned} \frac{dV_a}{dz} + \frac{dV_b}{dz} &= \left[\alpha - \frac{1}{2}\right]\left[\left(\frac{k_a}{w_a}\right)^{1+\gamma} - \left(\frac{k_b}{w_b}\right)^{1+\gamma}\right] z^{\gamma} \\ &= [1 - \xi(x, z, \boldsymbol{\iota})]\left[\alpha\left(\frac{k_a}{w_a}\right)^{1+\gamma} + (1-\alpha)\left(\frac{k_b}{w_b}\right)^{1+\gamma}\right] z^{\gamma}. \quad (15) \end{aligned}$$

For $\alpha < 1/2$, it is not hard to see that $\xi(x, z, \boldsymbol{\iota}) \geq (<)\, 1$ if and only if $k_a/w_a \leq (>)\, k_b/w_b$. In words, there is a utilitarian gain in increasing z whenever the low-power spouse ($\alpha < 1/2$) works less than the high-power spouse. The opposite is

[23] When preferences are separable, $\xi(x, z, \boldsymbol{\iota})$ does not depend on x. For the special case of identical iso-elastic preferences, it does not depend on z.

true, i.e., there is a utilitarian gain from reducing z, when she works more.[24]

The decomposition in the first line of (15) helps us to understand the two relevant forces that define ξ. The first term indicates how the planner's preferences differ from those of the household. It, therefore, indicates the direction toward which distribution is desirable. The second term determines whether it is by increasing or reducing z that distribution goes in the desired direction. If $\alpha = 1/2$, then there is no dissonance. The objectives of the household and the planner are perfectly aligned. When $\alpha/(1-\alpha) = w_a/w_b$, in contrast, the planner would want to transfer utility from one spouse to the other, but cannot do it with the available tax instruments.

Figure 2 displays the interaction between the two forces that determine the modified tax formula presented in this paper: the bargain weight (α) and the within-household effort gap. The top panel isolates the welfare transfer attained from a slight increase in z (at $z = 1$) as a function of α for different values of w_b, holding $w_a = 1$. It is indeed a graphic representation of equation (15). The bottom panel interacts with the term displayed in the top panel the planner's objective. When there is a misalignment between the planner and the household's objectives ($\alpha \neq 0.5$), the welfare transfer depends on whether the spouse that the planner wants to help is working more or less than the spouse from whom the utility transfer is desired.

The logic above applies to each specific family ι. To use it in (13), we must aggregate across families that share the same θ (or, in the absence of kinks in the tax schedule, the same z). This involves not only assessing the mean values for the two terms in the first line of (15) at each z but also their covariance.

5 Quantitative Assessment

We now compute the marginal tax rate derived in Proposition 1 using real data from the March 2016 extract of the CPS. This exercise will provide a quantitative assessment of how family decision-making affects the size and shape of optimal joint income taxes. In Appendix B, we have a detailed exposition of the implementation based on a discrete grid of the income distribution adapted from Mankiw et al. (2009) to our model.

5.1 Numeric Exercise

We restrict our sample to married spouses whose incomes are strictly positive. As in Mankiw et al. (2009), we use hourly wages as a proxy for productivity. The

[24]The gains are by reversing the direction in each case when $\alpha \geq 1/2$.

parameter γ in the disutility of labor is set to be equal to 2, implying a Frisch elasticity of labor supply of 0.5, as in Chetty (2009). Recall that a household type is $\boldsymbol{\iota} = (w_a, w_b, \alpha)$. We get the joint distribution of w_a and w_b from the data and we assume for our numeric exercise that

$$\alpha = \frac{w_a \varepsilon}{w_a \varepsilon + w_b(1-\varepsilon)}, \tag{16}$$

where ε is a random variable uniformly distributed, i.e., $\epsilon \sim U(0.2, 0.8)$.[25] This assumption allows us to capture the evidence uncovered in Browning and Chiappori (1998); Lise and Yamada (2019) that the spouses' productivity ratio affects their relative weight.[26]

From the distribution of $\boldsymbol{\iota}$, $F(\boldsymbol{\iota})$, we obtain the distribution, $\Psi(\theta)$, and the associated density, $\psi(\theta)$, aggregating across $\boldsymbol{\iota} \in \mathcal{I}(\theta)$, according to the definition given in (10). Taking a simple mean from CPS data and fitting it to structural equation (16), we find an average value of 0.46 for α, which is remarkably close to the estimates in Lise and Yamada (2019). Finally, for every $\boldsymbol{\iota} = (w_a, w_b, \alpha)$ in our sample we calculate $\xi(x, z, \boldsymbol{\iota})$, which, as we have discussed in Subsection 4.1, does not depend on (x, z), given our preference specification. Next, for every θ we calculate $\bar{\xi}(\theta)$, as defined in (11), by averaging across all $\boldsymbol{\iota}$ such that $\boldsymbol{\iota} \in \mathcal{I}(\theta)$. Because $\bar{\xi}(\theta)$ is independent of (x, z), the Pigouvian term in (13) only depends on the optimal allocation through the ratio $\mathbb{E}^{\psi}[x]/x(\theta)$. This simplifies our numerical procedure as we explain in Appendix B.

Concretely, for purposes of policy application, one might be interested in understanding which part of the spectrum of Figure 2 the economy is. For each family, ι, Table 1 summarizes the parameter configuration that pins down the sign of the right-hand side of the equation (15). We use the CPS data to compute for each couple in our sample the right-hand side of the equation (15) (without the endogenous term z).

Figure 4 plots this information using a dot for each household. Two things are apparent from the fitting line. First, most values and the adjusted line live in the negative half of the graph, showing that most observations are along the main

[25]We avoid the corner solutions for ϵ close to one and zero, which seems to be the more realistic case. Moreover, under this assumption

$$\frac{w_b}{1-\alpha}\frac{\alpha}{w_a} = \frac{\epsilon}{1-\epsilon},$$

which is a convex function of ϵ. For $\epsilon \sim U(1/2-\delta, 1/2+\delta)$, $\delta \in (0, 1/2)$, spouse b works more, $n_b > n_a$, on average.

[26]Specifically, Lise and Yamada (2019) find that cross-sectional differences at the time of marriage in expected wage profiles between a husband and wife affect the allocation of private consumption expenditures and time use by households in the cross-section.

Signing $dV_a/dz + dV_b/dz$

	$\frac{w_a}{w_b} < \frac{\alpha}{1-\alpha}$	$\frac{w_a}{w_b} > \frac{\alpha}{1-\alpha}$
$\alpha > 1/2$	−	+
$\alpha < 1/2$	+	−

Table 1: The minus (plus) sign indicates that decreasing (increasing) earnings is welfare improving.

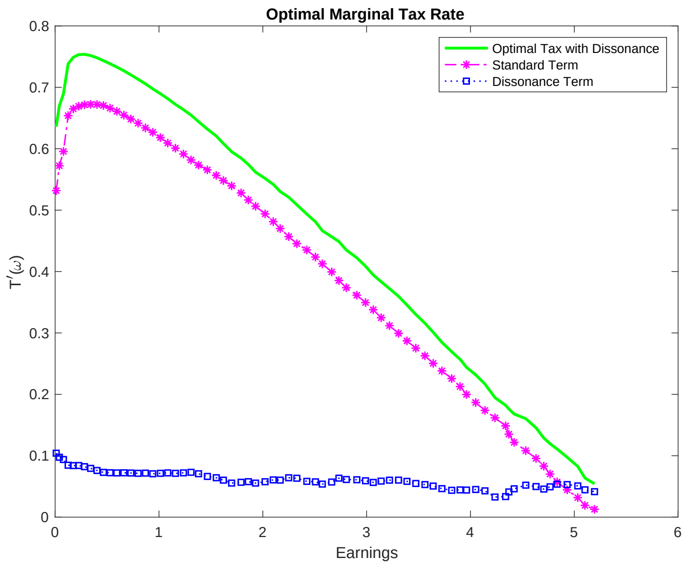

Figure 3: The solid green line depicts the values for $T'/(1-T')$ satisfying (13) in Proposition 1. The dotted lines decompose $T'/(1-T')$ into a standard Mirrleesian term (pink with diamond markers) and a novel modified part (blue with square markers) induced by dissonance.

diagonal of Table 1. Second, the values are more dispersed for more affluent families. As Figure 2 (top panel) shows, when spouse b is more productive (*triangle markers*), the utility transfer from a to b increases with α (bargaining power for the least productive spouse). The reason is that spouse b is more likely to work more as α increases. Whether this transfer is desirable, on the other hand, depends on whether $\alpha > 1/2$ – bottom panel. This is more clearly exemplified by the curve $w_b = 1$. Indeed, in this case, it is always the spouse with less power that works more. It is then always optimal for the planner to disincentivize work (increase marginal tax rates). The intensity of the positive change in marginal tax rates depends on the intra-household power gap.

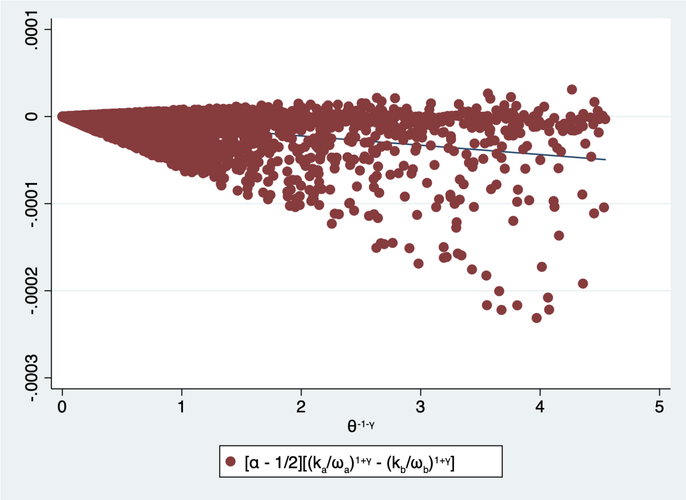

Figure 4: This figure shows the scatter plot for equation (15). Each dot represents a household and the fitting line shows that on average wealthier households present more inequality across spouses.

6 Extensions

6.1 Generalizing Preferences and Perturbation Methods

In the previous sections, we fully characterized the optimal tax schedules when spouses share the same iso-elastic preferences. It is possible to offer a characterization for the case of general preferences using a tax perturbation approach. The procedure, standard by now, is to compute the welfare and revenue effects of perturbing the baseline schedule in the direction of an arbitrary reform $H : \mathbb{R}_+ \to \mathbb{R}$, which amounts to computing its Gateaux differential, then noting that at the optimum, there is no direction of improvement.

Consider, therefore, a reform whereby the current schedule is replaced by a new one, $\tilde{T} = T + \mu H$, for $\mu \in \mathbb{R}$. When the reform H is a $\mathcal{C}^2$ function, for small values of μ the resulting tax schedule is a *perturbed* version of the baseline tax and lies within its neighborhood. This section restricts taxes to the class of joint taxation $\mathcal{T}_0$. Given a baseline tax system, $T \in \mathcal{T}_0$, we say that the perturbation $H : \mathbb{R}_+ \to \mathbb{R}$ is *within the same class* if the perturbed tax schedule remains in the same class as T, that is, $\tilde{T} = T + \mu H \in \mathcal{T}_0$ for small enough μ. We will then say that the tax schedule is optimal within this class if there is no perturbation *within the same class*, which improves the planner's objective.[27,28]

We use the standard definitions of uncompensated, $\varepsilon(z, \boldsymbol{\iota})$, compensated, $\varepsilon^c(z, \boldsymbol{\iota})$,

[27] There may still be H' for which $\tilde{T}' = T + \mu H' \notin \mathcal{T}_0$, which improves the planner's objective.

[28] Alves et al. (2019) show that formulae derived using variational methods may fail to characterize the optimum in models in which marginal rates of substitution are not exogenously ordered according to the unobserved heterogeneity, which typically arises in models with multidimensional heterogeneity. At issue is the implicit assumption that agents are never indifferent between two choices z and z' with $z > z'$ and, hence, that perturbations induce no jumps. Therefore, unless we impose more structure on agents' preferences, our model is susceptible to Alves et al.'s (2019) critique.

income, $\eta(z, \boldsymbol{\iota})$, and cross-sectional, $\gamma(z, \boldsymbol{\iota})$, elasticities of taxable income at earnings z for a household $\boldsymbol{\iota}$ – see Appendix C. We also let $\Phi(z)$ denote the distribution of z induced by the tax system and $\varphi(z)$, the associated density. Finally, using $g(z)$ to denote the average social welfare weight among all households that choose z, we derive in the online Appendix D, Proposition 3, an optimal formula under unrestricted preferences. This general expression allows for heterogeneous elasticities and welfare weights at each level of earnings, z. For Proposition 2 below, we restrict our attention to the case in which all couples, $\boldsymbol{\iota}$, choosing the same z, have the same elasticities. This simplifies the expression sufficiently for it to become easily comparable with our findings in Section 4.

Proposition 2. *Assume that $\varepsilon(z, \boldsymbol{\iota}) = \varepsilon(z)$, $\eta(z, \boldsymbol{\iota}) = \eta(z)$, and $\varepsilon^c(z, \boldsymbol{\iota}) = \varepsilon^c(z)$ for all $\boldsymbol{\iota}$ choosing z and for all z. Then, the marginal tax rate for the optimal joint tax schedule for couples at income level z is given by*

$$\frac{T'(z)}{1 - T'(z)} = A(z)B(z)C(z) + D(z), \tag{17}$$

where

$$A(z) = \frac{1}{\varepsilon^c(z)}, \quad B(z) = \frac{1 - \Phi(z)}{\varphi(z)z}, \quad D(z) = -g(z)\left[1 - \xi(z)\right]$$
$$\text{and} \quad C(z) = \int_z^{\bar{z}} \{1 - g(\tilde{z})\} \exp\left\{\int_z^{\tilde{z}} \frac{\eta(\tilde{\tilde{z}})}{\varepsilon^c(\tilde{\tilde{z}})} \frac{d\tilde{\tilde{z}}}{\tilde{\tilde{z}}}\right\} \frac{\varphi(\tilde{z})}{1 - \Phi(z)} d\tilde{z}.$$

Equation (17) has an ABC (and D) form similar to that in Diamond (1998), with an added D to account for a dissonance term. The term $D(z)$ acts as a Pigouvian correction that raises marginal tax rates when $\bar{\xi}(z) > 1$ and lowers it otherwise. Recalling that

$$\xi(z, \boldsymbol{\iota}) \lesseqqgtr 1 \Leftrightarrow \frac{\partial W(z - T(z), z, \boldsymbol{\iota})/\partial z}{\partial W(z - T(z), z, \boldsymbol{\iota})/\partial x} \lesseqqgtr \frac{\partial V(z - T(z), z, \boldsymbol{\iota})/\partial z}{\partial V(z - T(z), z, \boldsymbol{\iota})/\partial x},$$

marginal tax rates are reduced when compared to the pure Mirrleesian optimum if, on average, a family earning z underestimates the disutility of effort as perceived by the planner and are increased otherwise. For the case of iso-elastic preferences of Section 4, $\xi(z, \boldsymbol{\iota})$ is independent of z.[29]

6.2 Combining joint and individual instruments

Kleven et al. (2009) are perhaps the primary reference for deriving optimal tax

[29] Of course $\boldsymbol{\iota}$ will determine, along with the tax system, how much earnings, z, the family will generate. This means that knowledge about $\boldsymbol{\iota}$ is sufficient for determining ξ, for any tax system.

schedules for couples under a mechanism design approach. The complexities related to the multidimensional nature of the problem are circumvented by assuming that primary earners make choices along the intensive margin and secondary earners have identical labor market productivities and make choices along the extensive margin only. By introducing the notion of jointness in tax schedules, they explore the usefulness of making the marginal tax rates depend on whether the second earner participates in the labor market. Hence, they start from a tax system in which one spouse's marginal tax rate is independent of the other's choices and ask how the introduction of some dependence can improve outcomes.[30]

In this paper, we start from the opposite perspective: tax schedules are characterized by income-splitting, for which marginal tax rates are always equalized across spouses. In this section, we take a small step towards distinguishing taxes for the two spouses by slightly reducing the marginal tax rate of spouse a at all levels of income. That is, we discuss the consequences of adding a low tax, t, on the gross income of spouse a. Assume that we want to improve all spouse a's situation by offering them a linear (possibly negative) earnings' tax $t \in \mathbb{R}$. In this case, the household's budget constraint becomes

$$x_a + x_b \leq z_a(1-t) + z_b - T(z_a(1-t) + z_b).$$

Let $\hat{z}_a \equiv z_a(1-t)$ and $\hat{z} \equiv \hat{z}_a + z_b$. Then, we can write the families' optimal choice of earnings as the solution to

$$\min_{\hat{z}_a} \left\{ \alpha \left(\frac{\hat{z}_a}{\hat{w}_a} \right)^{1+\gamma} + (1-\alpha) \left(\frac{\hat{z} - \hat{z}_a}{w_b} \right)^{1+\gamma} \right\},$$

for $\hat{w}_a \equiv w_a(1-t)$ followed by the maximization with respect to $\hat{z}$. It is not hard to check that with $\ln$ preferences

$$\hat{k}_a(\boldsymbol{\iota}) = \frac{\hat{w}_a^{\frac{1+\gamma}{\gamma}} (1-\alpha)^{1/\gamma}}{\hat{w}_a^{\frac{1+\gamma}{\gamma}} (1-\alpha)^{1/\gamma} + w_b^{\frac{1+\gamma}{\gamma}} \alpha^{1/\gamma}} \text{ and } \hat{k}_b(\boldsymbol{\iota}) = 1 - \hat{k}_a(\boldsymbol{\iota}),$$

whereas the new disutility parameter becomes

$$\hat{\theta}(\boldsymbol{\iota}) := \hat{w}(\boldsymbol{\iota})^{-1-\gamma}, \quad \text{for} \quad \hat{w}(\boldsymbol{\iota}) := \left[\alpha^{\frac{-1}{\gamma}} \hat{w}_a^{\frac{1+\gamma}{\gamma}} + (1-\alpha)^{\frac{-1}{\gamma}} w_b^{\frac{1+\gamma}{\gamma}} \right]^{\frac{\gamma}{1+\gamma}}.$$

To understand the consequences of this policy for the distribution of utility across spouses one needs only to examine (14), which, after one introduces t, be-

[30]They calculate the optimum as well, but this is an important exercise they conduct to build the intuition for their findings.

comes

$$V_a - V_b = \ln \frac{\alpha}{1-\alpha} - \left[\left(\frac{\hat{k}_a}{\hat{w}_a} \right)^{1+\gamma} - \left(\frac{\hat{k}_b}{w_b} \right)^{1+\gamma} \right] \frac{\hat{z}^{1+\gamma}}{1+\gamma}. \tag{18}$$

For a fixed α, the reform cannot influence the relative consumption of spouses. Hence, the consequence of introducing a subsidy ($t < 0$) on a's depends only on how it affects the second term in the right-hand side of (18). A first consequence of this small subsidy is to induce an increase in z by increasing $\hat{w}(\boldsymbol{\iota})$. This transfers utility from or to the wife depending on whether w_a/w_b is less or equal to $\alpha/(1-\alpha)$, an effect that we have already discussed. Of course, this could have been accomplished more efficiently by simply changing $T(\cdot)$. The novel consequence arises through the impact on $(\hat{k}_a/\hat{w}_a)^{1+\gamma} - (\hat{k}_b/w_b)^{1+\gamma}$. For a large set of parameter values, subsidizing the secondary earner while keeping z fixed will transfer utility away from her to her spouse; a counter-intuitive, non-intended result.[31]

Policy-dependent α. The counter-intuitive result that subsidizing a spouse's earnings reduces her utility hinges on the assumption that the reform has no impact on α. Immervoll et al. (2011), who make the same independence assumption regarding the Pareto weights, justify it by assuming that α depends on the utilities attained by singles, which, they claim, are unaffected by the taxation of couples. One must note, however, that if power is determined by the 'shadow of marriage markets', as suggested by Becker (1973, 1974), then we need to be explicit about how marriage markets are affected by the reform to be able to make such statements. Instead of taking this path we just offer a simple example of how the results are easily overturned if we allow the policy to affect spouses' power within the collective framework.

The example is based on an additional potential impact of subsidizing a: to increase her bargaining power by increasing her (net) relative earnings potential. Indeed, in our baseline implementation, we assumed $\alpha = w_a\epsilon/(w_a\epsilon + w_b(1-\epsilon))$ where ϵ is a random variable in $(0, 1)$. This generates the empirically plausible association of power with relative earnings potential. Let us for the moment assume that ϵ is a degenerate random variable that takes the value 0.5 with probability one. In this case, $\hat{\alpha} = \hat{w}_a/(\hat{w}_a + w_b)$ and the term in brackets in (18) does not depend on t. Subsidizing a's earnings only affects her by transferring consumption from b to a, restoring the intuitive result.

The purpose of this discussion is not to rule out the counter-intuitive result, but simply to draw attention to the fact that the theory underlying most findings

[31] A sufficient condition for this counter-intuitive result in our setting is $\alpha > (\gamma-1)/2\gamma$ – see the online Appendix E.

in optimal taxation of couples is not compatible with some gender-based policies that have been advocated and adopted in many parts of the world, in the sense that the latter cannot be rationalized by a fixed α approach.

7 Conclusion

In this paper, we obtain optimal tax schedules for multiperson households under the restriction that taxes entail income-splitting. The use of a commonly adopted specification for preferences allows us to handle the multidimensional screening problem that plagues optimal household taxation and to actually compute the optimal tax schedule. Our results highlight the quantitative impact of dissonance, the misalignment between the household's and planner's objectives for household income taxes: an instrument that can only indirectly affect intrahousehold inequality. Optimal marginal tax rates need no longer be non-negative which is in contrast with a well-known property derived for single-person households in the Mirrlees tradition.

As an extension, we explore the consequences of introducing some form of differentiation in the marginal tax rates faced by the two spouses. We show that it is typically optimal to introduce a *small* tax on the spouse that society tries to promote since income taxes subsidize the spouse's leisure. This crucially depends on the assumption that power is unaffected by choices or the tax system (or both). It is clear that if earnings – a policy-dependent variable – instead of innate productivity – a policy-independent variable – determine power, these results may be reversed.

In our model, all effort is directed to the labor market, whereas in practice, low-power agents often dedicate a substantial amount of time to home production, which is something our model does not contemplate. For the most reasonable specifications of home production, its inclusion would break the aggregation result that allows us to apply a mechanism design approach to our model, which is why we have refrained from doing it in this paper. Still, we may speculate about the likely effects of considering it.

An increase in marginal tax rates would have the added effect of inducing spouses to do more home production. With home production, secondary would most likely spend a larger fraction of her working time on home production. A crucial thing to consider would be whether it would be the spouse who was already involved in most home production or her spouse that would increase home production the most. Recall that with market work only and identical iso-elastic preferences, relative effort on average is identical to relative effort at the margin, but, with home production, this would not be the case, in general. Still, a plausible scenario is one in which an increase in the marginal tax rate would increase home production and the relative effort of the secondary earner. Working out the details of such a model, even if only a parametric solution to the tax problem were feasible, is a very interesting path for future work.

References

Alm, J. and L. A. Whittington (1999). For love or money? the impact of income taxes on marriage. *Economica 66*(263), 297–316. 12

Alves, C. B., C. E. da Costa, and H. Moreira (2019). Tax perturbation without the spence-mirrlees' condition: The role of income segregation. 24, D-7

Alves, C. B. M., C. E. d. Costa, and H. A. Moreira (2021). Intrahousehold inequality and the joint taxation of household earnings. FGV EPGE Economics Working Papers (Ensaios Economicos da EPGE) 825, EPGE Brazilian School of Economics and Finance - FGV EPGE (Brazil). 17

Anderson, S. and D. Ray (2010, 10). Missing Women: Age and Disease. *The Review of Economic Studies 77*(4), 1262–1300. 2

Apps, P. and R. Rees (1988). Taxation and the household. *Journal of Public Economics 35*, 355–369. 1, 2, 7, 8, 14

Bastani, S. (2013). Gender-based and couple-based taxation. *International Tax and Public Finance 20*(4), 653–686. 7

Becker, G. S. (1973). A theory of marriage (part i). *Journal of Political Economy 81*, 813–846. 27

Becker, G. S. (1974). A theory of marriage (part ii). *Journal of Political Economy 82*, 511–526. 27

Blundell, R., L. Pistaferri, and I. Saporta-Eksten (2016). Consumption inequality and family labor supply. *American Economic Review 106*(2), 387 – 435. 8

Boskin, M. J. and E. Sheshinski (1983). Optimal tax treatment of the family: Married couples. *Journal of Public Economics 20*, 281–297. 2, 7

Browning, M. and P. Chiappori (1998). Efficient intra-household allocation: a characterization and tests. *Econometrica 66*(6), 1241–1278. 12, 22

Calvi, R. (2020). Why are older women missing in india? the age profile of bargaining power and poverty. *Journal of Political Economy 128*(7), 2453–2501. 2

Chavas, J.-P., M. Menon, E. Pagani, and F. Perali (2018). Collective household welfare and intra-household inequality. *Theoretical Economics 13*(2), 667–696. 2

Chetty, R. (2006). A new method for estimating risk aversion. *American Economic Review 96*, 1821–1834. 15

Chetty, R. (2009). Sufficient statistics for welfare analysis: a bridge between structural and reduced form methods. *Annual Review of Economics 1*(1), 451 – 488. 22

Chiappori, P. (1988a). Nash-bargained household decisions. *International Economic Review 32*, 791–796. 12

Chiappori, P. (1988b). Rational household labor supply. *Econometrica 56*, 63–89. 2, 7

Chiappori, P. (1992). Collective labor supply and welfare. *Journal of Political Economy 100*, 437–467. 2, 7

Chiappori, P.-A. and C. Meghir (2015). Intrahousehold inequality. In A. B. Atkinson and F. Bourguignon (Eds.), *Handbook of Income Distribution*, Volume 2 of *Handbook of Income Distribution*, pp. 1369 – 1418. Elsevier. 2, 13

Choné, P. and G. Laroque (2010, December). Negative marginal tax rates and heterogeneity. *American Economic Review 100*(5), 2532–47. 7, 17

Cremer, H., J.-M. Lozachmeur, D. Maldonado, and K. Roeder (2016). Household bargaining and the design of couples' income taxation. *European Economic Review 89*, 454–470. 4, 8

Cremer, H., J.-M. Lozachmeur, and P. Pestieau (2012). Income taxation of couples and the tax unit choice. *Journal of Population Economics 25*(2), 763–778. 2, 8, 10

da Costa, C. E. and L. A. de Lima (2019). Mechanism design and the collective approach to household behavior. Mimeo FGV EPGE. 12

Diamond, P. A. (1998). Optimal income taxation: an example with a u-shaped pattern of optimal marginal tax rates. *American Economic Review 88*(1), 83–95. 9, 25

Farhi, E. and X. Gabaix (2020, January). Optimal taxation with behavioral agents. *American Economic Review 110*(1), 298–336. 14

Gayle, G. and A. Shephard (2019). Optimal taxation, marriage, home production, and family labour supply. *Econometrica 87*(1), 291–326. 12

Gerritsen, A. (2016). Optimal taxation when people do not maximize well-being. *Journal of Public Economics 144*, 122–139. 4, 8, 9, 14, D-11

Gordon, R. and W. Kopczuk (2014). The choice of personal income tax base. *Journal of Public Economics 118*, 97 – 110. 4

Guner, N., R. Kaygusuz, and G. Ventura (2012). Taxation and household labour supply. *The Review of Economic Studies 79*(3), 1113–1149. 8

Haddad, L. and R. Kanbur (1990). How serious is the neglect of intra-household inequality? *The Economic Journal 100*(402), 866–881. 2

Heathcote, J., K. Storesletten, and G. L. Violante (2017). Optimal Tax Progressivity: An Analytical Framework*. *The Quarterly Journal of Economics 132*(4), 1693–1754. 17

Heathcote, J. and H. Tsujiyama (2021). Optimal income taxation: Mirrlees meets ramsey. *Journal of Political Economy 129*(11), 3141–3184. 3, 7

Immervoll, H., H. Kleven, C. Kreiner, and N. Verdelin (2011). Optimal tax and transfer programs for couples with extensive labor supply responses. *Journal of Public Economics 95*(11), 1485–1500. 4, 8, 11, 27

Jacquet, L., E. Lehmann, and B. Van der Linden (2013). Optimal redistributive taxation with both extensive and intensive responses. *Journal of Economic Theory 148*(5), 1770–1805. C-5, D-9

Kleven, H. J., C. T. Kreiner, and E. Saez (2009). The optimal income taxation of couples. *Econometrica 77*(2), 537–560. 2, 4, 7, 25

Lise, J. and K. Yamada (2019). Household sharing and commitment: Evidence from panel data on individual expenditures and time use. *Review of Economic Studies 86*(5), 2184–2219. 22

Lundberg, S. and R. A. Pollak (1996). Bargaining and distribution in marriage. *The Journal of Economic Perspectives 10*(4), 139–158. 7

Mankiw, N. G., M. Weinzierl, and D. Yagan (2009). Optimal taxation in theory and practice. *The Journal of Economic Perspectives 23*(4), 147–174. 21, B-3

Manser, M. and M. Brown (1980). Marriage and household decision making: a bargaining analysis. *International Economic Review 21*(1), 31–44. 7

McElroy, M. B. and M. J. Horney (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International Economic Review 22*(2), 333–349. 7

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *The Review of Economic Studies 38*(2), 175–208. 1, 2, 5, 6, 11, 16, 18, 19

Piketty, T. (1997). La redistribuition fiscale face au chômage. *Revue Française d' Èconomie 12*, 157 – 201. D-10

Rochet, J. and L. Stole (1987). The economics of multidimensional screening. In M. Dewatripont, L. P. Hansen, and S. J. Turnovsky (Eds.), *Advances in Economics and Econometrics: Theory and Applications - Eight World Congress*, pp. 150–198. Cambridge University Press. 11

Rochet, J.-C. and P. Choné (1998). Ironing, sweeping, and multidimensional screening. *Econometrica 66*(4), 783–826. 11

Rochet, J.-C. and L. A. Stole (2003). The economics of multidimensional screening. *Econometric Society Monographs 35*, 150–197. 2

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies 68*, 205–229. 4, 8, C-5, D-8, D-10, D-11, D-12, D-13

Scheuer, F. and I. Werning (2017). The taxation of superstars. *The Quarterly Journal of Economics 132*(1), 211–270. C-5, D-9

Schumpeter, J. A. (1954). *History of Economic Analysis*. Routledge. 2

Tuomala, M. (2016). *Optimal redistributive taxation*. Oxford University Press. 15

Wu, C. and D. Krueger (2021). Consumption insurance against wage risk: Family labor supply and optimal progressive income taxation. *American Economic Journal: Macroeconomics 13*(1), 79–113. 8

A Proofs

Proof of Proposition 1. The planner's program is

$$\max_{(x(\theta),z(\theta))_\theta} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \bar{V}(x(\theta), z(\theta), \theta) + \left[1 - \bar{\xi}(\theta)\right] \theta \frac{z^{1+\gamma}}{1+\gamma} \right\} \psi(\theta) d\theta, \tag{A.1}$$

subject to

$$\theta \in \operatorname*{argmax}_{\hat{\theta}} V(x(\hat{\theta}), z(\hat{\theta}), \theta)$$

and

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[z(\theta) - x(\theta)\right] \psi(\theta) d\theta \geq G. \tag{A.2}$$

For all θ, let $v(\theta) = \max_{\hat{\theta}} V(x(\hat{\theta}), z(\hat{\theta}), \theta)$ and define $x(v(\theta), z(\theta), \theta)$ by

$$v(\theta) \equiv V(x(v(\theta), z(\theta), \theta), z(\theta), \theta).$$

Then, the incentive compatibility constraint is equivalent to

$$\dot{v}(\theta) = \frac{\partial}{\partial \theta} V(x(\theta), z(\theta), \theta), \tag{A.3}$$

and $z(\theta)$ non-increasing in θ.

Ignoring, as usual, the monotonicity constraint, the planner's program is to maximize (A.1) subject to (A.2) and (A.3). The Hamiltonian associated with this problem is

$$\mathcal{H}(v(\theta), z(\theta), \mu(\theta), \lambda, \theta) = \Big\{ v(\theta) + \left[1 - \bar{\xi}(\theta)\right] \theta \frac{z(\theta)^{1+\gamma}}{1+\gamma} + \lambda \left[z(\theta) - x(v(\theta), z(\theta), \theta)\right] \Big\} \psi(\theta) + \mu(\theta) \frac{z(\theta)^{1+\gamma}}{1+\gamma},$$

where $\lambda \in \mathbb{R}_+$ is the Lagrange multiplier associated with the government budget constraint and $\mu(\theta)$ is the co-state variable. At the optimum, the control $z(\theta)$ maximizes the Hamiltonian function giving the following first-order condition:

$$\left[1 - \bar{\xi}(\theta)\right] \theta z(\theta)^\gamma \psi(\theta) + \lambda \left[1 - \frac{\partial x(v(\theta), z(\theta), \theta)}{\partial z}\right] \psi(\theta) = -\mu(\theta) z(\theta)^\gamma.$$

Noting that $\frac{\partial x(v(\theta),z(\theta),\theta)}{\partial z} = 1 - T'(z(\theta))$ and $\theta z(\theta)^\gamma = (1 - T'(z(\theta)))/x(\theta)$, we get

$$\left[1 - \bar{\xi}(\theta)\right] [1 - T'(z(\theta))] \psi(\theta) + \lambda T'(z(\theta)) x(\theta) \psi(\theta) = -\mu(\theta) [1 - T'(z(\theta))]/\theta,$$

which can be re-arranged to read

$$\frac{T'(z(\theta))}{1-T'(z(\theta))} = \frac{1}{\lambda x(\theta)\psi(\theta)\theta}\mu(\theta) - \frac{1}{\lambda x(\theta)}\left[1-\bar{\xi}(\theta)\right].$$

As for $\mu(\theta)$, the other first-order condition of the Hamiltonian is

$$\dot{\mu}(\theta) = -\frac{\partial}{\partial v}\mathcal{H}(\cdot) = \left[\lambda\frac{\partial x(v(\theta),z(\theta),\theta)}{\partial v} - 1\right]\psi(\theta)$$

and the transversality conditions $\mu\left(\underline{\theta}\right) = \mu\left(\overline{\theta}\right) = 0$. Integrating on both sides from $\underline{\theta}$ to θ and applying the fundamental theorem of calculus we have

$$\mu\left(\theta\right) - \mu(\underline{\theta}) = \int_{\underline{\theta}}^{\theta}\dot{\mu}(s)ds = \int_{\theta}^{\bar{\theta}}\left[\lambda\frac{\partial x(v(s),z(s),s)}{\partial v} - 1\right]\psi(s)ds.$$

Using the transversality condition we have

$$\mu(\theta) = \lambda\int_{\underline{\theta}}^{\theta}\left[\frac{\partial x(v(s),z(s),s)}{\partial v} - \lambda^{-1}\right]\psi(s)ds. \tag{A.4}$$

Note that from the transversality condition $\mu(\underline{\theta}) = 0$ we can pin down the social value of an extra unit of income:

$$0 = \mu(\underline{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}}\left[\frac{1}{\lambda} - \frac{\partial x(v(\theta),z(\theta),\theta)}{\partial v}\right]\psi(\theta)d\theta.$$

Therefore,

$$\lambda = \left\{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial x(v(\theta),z(\theta),\theta)}{\partial v}\psi(\theta)d\theta.\right\}^{-1}$$

which, due to $\partial x(v(\theta),z(\theta),\theta)/\partial v = x(v(\theta),z(\theta),\theta)$, for all θ, yields $\lambda = \mathbb{E}^{\psi}[x]^{-1}$.

□

B Implementation Algorithm for the Numeric Simulations

In this section, we explain how we compute numerically the optimal tax schedules derived in Proposition 1 using real data from the 2017 March CPS to have a clear idea of the shape of the optimal taxation in both cases. We use the sample of couples without dependents, using wages as a proxy for individual productivity.

For each (w_a, w_b) we generate $\boldsymbol{\iota}$ by making a draw ε from a uniform distribution in the interval $[0.2, 0, 8]$ and defining α using (16). This procedure gives us the primitive distribution $F(\boldsymbol{\iota})$ from which we define $\Phi(\theta)$ by $\Phi(\theta) = 1 - F(\boldsymbol{\iota}|\boldsymbol{\iota} \in \mathcal{I}(\theta'); \theta' \leq \theta)$.

After calculating these objects, we simulate the marginal tax rate by adapting the algorithm proposed by Mankiw et al. (2009), based on a discrete grid of the empirical distributions. As it turns, it is both simpler to adapt Mankiw et al.'s algorithm and easier to communicate our results if we use $w = \theta^{-1-\gamma}$ instead of θ and for all w define $\Psi(w) = 1 - \Phi(w^{-1-\gamma})$. With some abuse, let $\psi(w)$ denote the associated density. For every w we also calculate, again, with some abuse, $\bar{\xi}(w) = \bar{\xi}(w^{-1-\gamma})$ from its definition, (11).

Step 1: Given $\psi(w)$, discretize the probability mass function $\pi(w)$ in the following way: divide the interval considered $[\underline{w}, \overline{w}]$ in N bins of equal bandwidth Δ, creating a grid where $\{w_i\}_{i=1}^N$ with $\underline{w} \leq w_1 \leq \cdots \leq w_n \leq \cdots \leq w_N \leq \overline{w}$ are the midpoints in the grid. The probability mass function at each w is given by

$$\pi(w) = \Psi(w + \Delta/2) - \Psi(w - \Delta/2),$$

where $\Psi(\cdot)$ is the cumulative distribution function of w.

Step 2: For each $\{w_i\}_{i=1}^N$ calculate the discretized $\bar{\xi}(w_i)$ through

$$\bar{\xi}(w_i) = \frac{1}{\#\mathcal{I}(w_i)} \sum_{\boldsymbol{\iota} \in \mathcal{I}(w_i)} \xi(\boldsymbol{\iota}),$$

where we relied on ξ's being independent of (x, z).

Now we start the loop for the calculation of the optimal marginal tax rate $T_k(w_i)$.

Step 3: Start with a simple guess for the marginal tax rate. For instance, a flat tax schedule $T_0(w_i) = .35$ for all w_i in the grid and a lump-sum transfer $t_0 = .0001$.

Step 4: Since $T_{k-1}(\cdot)$ at each iteration is only calculated at points of the grid, extrapolate $T_{k-1}(\cdot)$ to be defined as well at points outside the grid in the following way:

$$T'_{k-1}(w) = \begin{cases} T'_{k-1}(w_1), \text{ for } \underline{w} \leq w \leq w_1; \\ T'_{k-1}(w_2), \text{ for } w_1 \leq w \leq w_2; \\ \cdots \\ T'_{k-1}(w_n), \text{ for } w_{n-1} \leq w \leq w_n; \\ \cdots \\ T'_{k-1}(w_N), \text{ for } w_{N-1} \leq w \leq \overline{w}; \end{cases}$$

generating the tax schedule

$$T_{k-1}(w_i) = \begin{cases} T'_{k-1}(w_1)z(w_i) - t_{k-1}, \text{ for } \underline{w} \leq w_i \leq w_1; \\ T'_{k-1}(w_1)z(w_1) + T'_{k-1}(w_2)[z(w_i) - z(w_1)] - t_{k-1}, \text{ for } w_1 \leq w_i \leq w_2; \\ \dots \\ T'_{k-1}(w_1)z(w_1) + \sum_{j=2}^{n-1} T'_{k-1}(w_j)[z(w_j) - z(w_{j-1})] + \\ \qquad T'_{k-1}(w_n)[z(w_i) - z(w_{n-1})] - t_{k-1}, \text{ for } \quad w_{n-1} \leq w_i \leq w_n; \end{cases}$$

Step 5: Given $T_{k-1}(\cdot)$, calculate the optimal labor supply at each w_i by solving

$$\max_z \ln(z - T_{k-1}(z)) - w_i \frac{z^{1+\gamma}}{1+\gamma}.$$

The necessary first-order condition for the type w_i problem is

$$1 - T'_{k-1}(z) = w_i z^{\gamma} (z - T_{k-1}(z)) \tag{B.1}$$

and the optimal $z^{\star}(w_i)$ is implicitly defined in this first-order condition.

Step 6: Given the optimal choice of $z(w_i)$, we can define the consumption as

$$x(w_i) = z^{\star}(w_i) - T_{k-1}(z^{\star}(w_i))$$

and use all the information to update the marginal tax rate at each point of the grid using

$$T'_k(w_i) = \frac{z^{\star}(w_i)^{\gamma}}{\psi(w_i)} \int_{w_i}^{\overline{w}} \left[x(s) - \mathbb{E}^{\psi}[x]\right] \psi(s) ds - \frac{\mathbb{E}^{\psi}[x]}{x(w_i)} \left[1 - \overline{\xi}(w_i)\right]. \tag{B.2}$$

Since the family utility satisfies the single-crossing property w.r.t. w_i, this first-order condition and the monotonicity of $z^{\star}(w_i)$ are sufficient for optimality.

Step 7: The formula in *Step 6* is unfeasible to estimate given our approximation of the wage distribution in the grid. Therefore, we must use a discrete approximation. One possibility is as follows:

$$T'_k(w_i) \approx \frac{1}{x(w_i)w_i} \frac{1}{\pi(w_i)/\Delta} \left[\sum_{j=i+1}^{N} \left[x(w_j) - \hat{\mathbb{E}}[x]\right] \pi(w_j)\right] - -\frac{\mathbb{E}^{\psi}[x]}{x(w_i)} \left[1 - \overline{\xi}(w_i)\right]. \tag{B.3}$$

where $\bar{\xi}(w_i)$ is calculated at *Step 2* and

$$\hat{\mathbb{E}}[x] = \sum_{j=1}^{N} x(w_j)\pi(w_j).$$

Step 8: Update t to ensure the budget constraint in the following way:

$$t_k = \sum_{j=1}^{N} T_k(w_j)\,\pi(w_j).$$

Step 9: Return to *Step 4* until

$$\max_i\{|T_k'(w_i) - T_{k-1}'(w_i)|\} < 10^{-3}.$$

Step 10: After getting a fixed point of this operator, the only thing to be checked is the monotonicity to guarantee that the found tax schedule is implementable.

Note that this approximation is increasingly precise as $\Delta \to 0$ and $w_N \to +\infty$.

C Elasticities

We first recall that in Saez's original paper, elasticities were defined for linearized budget sets. Here, we follow, instead, Jacquet et al. (2013) and Scheuer and Werning (2017) in using the elasticities defined under nonlinear budget sets. That is, we define the elasticity of taxable income with respect to the retention rate $1 - T'(z)$, $\epsilon(z, \boldsymbol{\iota})$, as

$$\epsilon(z,\boldsymbol{\iota}) = \Delta^{-1}\left[\partial_{xz}V(x,z,\boldsymbol{\iota})z - \partial_{xx}V(x,z,\boldsymbol{\iota})\frac{\partial_z V(x,z,\boldsymbol{\iota})}{\partial_x V(x,z,\boldsymbol{\iota})}z - \partial_x V(x,z,\boldsymbol{\iota})\right]\frac{1-T'(z)}{z},$$

where

$$\Delta = \Bigg[\partial_{xx}V(x,z,\boldsymbol{\iota})\left(\frac{\partial_z V(x,z,\boldsymbol{\iota})}{\partial_x V(x,z,\boldsymbol{\iota})}\right)^2 - 2\partial_{xz}V(x,z,\boldsymbol{\iota})\frac{\partial_z V(x,z,\boldsymbol{\iota})}{\partial_x V(x,z,\boldsymbol{\iota})} + \partial_{zz}V(x,z,\boldsymbol{\iota}) - \partial_x V(x,z,\boldsymbol{\iota})T''(z)\Bigg].$$

The last term in Δ, $\partial_x V(x,z)T''(z)$, captures the curvature in the budget set. Saez (2001), in contrast, defines the elasticity using $\tilde{\Delta} = \Delta + \partial_x V(x,z)T''(z)$. The advantage of using $\tilde{\Delta}$ is its familiarity, whereas the advantage of using Δ is the simplification of optimal tax formulae.

We can also define the income elasticity of taxable earnings

$$\eta(z, \boldsymbol{\iota}) = \Delta^{-1} \left[\partial_{xx} V(x, z, \boldsymbol{\iota}) \frac{-\partial_z V(x, z, \boldsymbol{\iota})}{\partial_x V(x, z, \boldsymbol{\iota})} + \partial_{xz} V(x, z, \boldsymbol{\iota}) \right] (1 - T'(z))$$

and, using the Slutsky equation, we can compute the compensated elasticity

$$\epsilon^c(z, \boldsymbol{\iota}) = \epsilon(z, \boldsymbol{\iota}) - \eta(z, \boldsymbol{\iota}) = \Delta^{-1} \left[\partial_x V(x, z, \boldsymbol{\iota}) \right] \frac{1 - T'(z)}{z}.$$

Online Appendix – not for publication

D Generalizing Preferences: The Tax Perturbation Approach

In the previous sections, we were able to fully characterize the optimal tax schedules when spouses share the same iso-elastic preferences. This section uses a tax perturbation approach to provide expressions for the optimal marginal tax rate for the case in which preferences are fully general.

The first step is to compute the welfare and revenue effects of perturbing the baseline schedule in the direction of an arbitrary reform $H : \mathbb{R}_+ \to \mathbb{R}$, which amounts to computing its Gateaux differential. That is, consider a reform whereby the current schedule is replaced by a new one, $\tilde{T} = T + \mu H$, for $\mu \in \mathbb{R}$. When the reform H is a $\mathcal{C}^2$ function, for small values of μ the resulting tax schedule is a *perturbed* version of the baseline tax and lies within its neighborhood. This section restricts taxes to the class of joint taxation $\mathcal{T}_0$. Given a baseline tax system, $T \in \mathcal{T}_0$, we say that the perturbation $H : \mathbb{R}_+ \to \mathbb{R}$ is *within the same class* if the perturbed tax schedule remains in the same class as T, that is, $\tilde{T} = T + \mu H \in \mathcal{T}_0$ for small enough μ. We will then say that the tax schedule is optimal within this class if there is no perturbation *within the same class*, which improves the planner's objective.[32,33]

We start by deriving the behavioral response for the change in a baseline tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$ (a candidate for the optimal schedule) in the direction of the reform $H : \mathbb{R}_+ \to \mathbb{R}$. Consider the program of a type $\boldsymbol{\iota}$ household facing a nonlinear tax schedule $T : \mathbb{R}_+ \to \mathbb{R}$:

$$\max_{(x,z)\in\mathbb{R}_+^2} \left\{ V(x, z, \boldsymbol{\iota}) \quad \text{s.t.} \quad x = z - T(z) \right\}. \tag{D.1}$$

The solution to this program defines the earnings supply functional $z_{\boldsymbol{\iota}} : \mathcal{T} \to \mathbb{R}$, where $\mathcal{T}$ is the class of tax schedules being considered. We measure *behavioral responses* to the reform in the direction of H by the Gateaux derivative of the earnings supply functional, denoted by $dz_{\boldsymbol{\iota}}(T; H)$ (see (D.7) for the formal definition).

[32] There may still be H' for which $\tilde{T}' = T + \mu H' \notin \mathcal{T}_0$, which improves the planner's objective. In fact, in Subsection 6.2 we study reforms whose resulting perturbed classes are not within the class of joint taxation $\mathcal{T}_0$.

[33] Alves et al. (2019) show that formulae derived using variational methods may fail to characterize the optimum in models in which marginal rates of substitution are not exogenously ordered according to the unobserved heterogeneity, which typically arises in models with multidimensional heterogeneity. At issue is the implicit assumption that agents are never indifferent between two choices z and z' with $z > z'$ and, hence, that perturbations induce no jumps. Therefore, unless we impose more structure on agents' preferences, our model is susceptible to Alves et al.'s (2019) critique.

The overall marginal effect on welfare after a reform in the direction of H is given by

$$dW(x,z,\boldsymbol{\iota}) = \partial_x W(x,z,\boldsymbol{\iota})\left[-H(z)+\left(1+\frac{\partial_z W(x,z,\boldsymbol{\iota})}{\partial_x W(x,z,\boldsymbol{\iota})}\frac{1}{1-T'(z)}\right)[1-T'(z)]dz_{\boldsymbol{\iota}}(T;H)\right]d\mu. \tag{D.2}$$

The first term in (D.2), represents the difference between a dollar in the hands of a household and a dollar in the hands of the government. The second term captures the consequences of behavioral responses for government revenues. Typically, this second term is zero due to the envelope condition. When there is dissonance, however, the envelope arguments normally used to capture the impact on the welfare of these perturbations (e.g., Saez (2001)) must therefore be amended to include this term.[34]

Let us derive expressions for $\partial_z W(x,z;\boldsymbol{\iota})/\partial_x W(x,z;\boldsymbol{\iota})$ and $dz_{\boldsymbol{\iota}}(T;h)$. To make our point in as stark a manner as possible and to better communicate with the numerical example of Section 4, we restrict our attention to separable preferences of the form $U(x_i,l_i)=u(x_i)-h(l_i)$ for functions $u:\mathbb{R}_+\to\mathbb{R}$ and $h:\mathbb{R}_+\to\mathbb{R}$. We then define, with some abuse of notation and omitting parameters for shortness, $x_a(x)$, $x_b(x)$, $z_a(z)$ and $z_b(z)$ as the solution to the family program in (2).[35] In this case, we can show that

$$\frac{\partial_z W(x,z,\boldsymbol{\iota})}{\partial_x W(x,z,\boldsymbol{\iota})}\frac{1}{1-T'(z)} = -\frac{(2\alpha-1)\,z_a'(z)+(1-\alpha)}{(2\alpha-1)\,x_a'(x)+(1-\alpha)}.$$

Of course, if $\alpha=1/2$, there is no dissonance and the marginal rate of substitution from the planner's perspective is equal to the marginal retention rate $1-T'(z)$. This is also true in the limiting case when $x_a'(x)=z_a'(z)$, but this result is unexpected.[36]

The behavioral response to a reform in the direction of H, measured by the Gateaux derivative, is given by

$$dz_{\boldsymbol{\iota}}(T;H) = -\frac{\varepsilon^c(z,\boldsymbol{\iota})zH'(z)+\eta(z,\boldsymbol{\iota})H(z)}{1-T'(z)+\varepsilon^c(z,\boldsymbol{\iota})zT''(z)}, \tag{D.3}$$

[34] It is easy to see the source of these differences. The first-order necessary conditions for program (D.1) is $1-T'(z)=-\partial_z V(x,z,\boldsymbol{\iota})/\partial_x V(x,z,\boldsymbol{\iota})$. In a standard Mirrleesian model, $W(x,z,\boldsymbol{\iota})=V(x,z,\boldsymbol{\iota})$, so that $1-T'(z)=-\partial_z W(x,z,\boldsymbol{\iota})/\partial_x W(x,z,\boldsymbol{\iota})$.

[35] Notice that separability of the utility implies separability between consumption and labor decisions. That is why we write $x_a(\cdot)$ and $x_b(\cdot)$ as a function only of x, and $z_a(\cdot)$ and $z_b(\cdot)$ as function only of z.

[36] While $x_a'(x)$ depends on the local curvature of $u(\cdot)$ at the optimal choice $(x_a(x),x-x_a(x))$, $z_a'(z)$ depends on the local curvature of $h(\cdot)$ at the optimal choice $(z_a(z),z-z_a(z))$. Hence, they will differ for almost all parametrizations.

where $\varepsilon^c(z,\boldsymbol{\iota})$ is the compensated elasticity of household taxable earnings with respect to the retention rate $1-T'(z)$, and $\eta(z,\boldsymbol{\iota})$ is the income elasticity of taxable earnings of a type $\boldsymbol{\iota}$, both of which are explicitly calculated in Appendix C.

The relevant elasticities For each household $\boldsymbol{\iota}$, we follow Jacquet et al. (2013) and Scheuer and Werning (2017) in deriving the relevant elasticities of taxable income that take into account the endogeneity of marginal tax rates due to the local tax schedule curvature: uncompensated, $\epsilon(z,\boldsymbol{\iota})$, income, $\eta(z,\boldsymbol{\iota})$, and compensated, $\epsilon^c(z,\boldsymbol{\iota})$ (see Appendix C).

All of these elasticities are defined at the household level. However, the tax schedule should necessarily treat all families with the same total income equally. Therefore, we need to aggregate across all families. For each $z \in \mathbb{R}_+$, let

$$\bar{\varphi}(\boldsymbol{\iota}|z) = \int_{z_{\boldsymbol{\iota}}(T)=z} dF(\boldsymbol{\iota})$$

be the distribution of household types $\boldsymbol{\iota}$ choosing z at the candidate optimal tax schedule T. We can aggregate across households choosing z using $\bar{\varphi}(\boldsymbol{\iota}|z)$ and define aggregate elasticities as follows:

$$\varepsilon(z) = \int_{\boldsymbol{\iota}\in\Lambda} \varepsilon(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z), \quad \eta(z) = \int_{\boldsymbol{\iota}\in\Lambda} \eta(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z) \quad \text{and} \quad \varepsilon(z)^c = \int_{\boldsymbol{\iota}\in\Lambda} \varepsilon^c(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z).$$

Let us denote

$$\Phi(z) = \int_{\boldsymbol{\iota}\in\Lambda} d\bar{\varphi}(\boldsymbol{\iota}|z)$$

as the empirical income distribution induced by the tax system, and let's assume it admits a density, which we shall denote by $\varphi(z)$.

As it turns out, elasticities $\varepsilon(z)$, $\eta(z)$, and $\varepsilon^c(z)$ are important for understanding the consequences of behavioral responses to government revenues. However, a novelty we uncover is that we need alternative elasticities to assess the welfare consequence of behavioral responses. For this, let the average welfare weight at income z, $g(z)$, be defined as

$$g(z) = \int_{\boldsymbol{\iota}\in\Lambda} g(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z),$$

where $g(z,\boldsymbol{\iota}) = \partial_x W(z-T(z),z,\boldsymbol{\iota})$, which is positive given our specification of the welfare function (3). Hence, we define the welfare-weighted elasticity by

$$\bar{\varepsilon}(z) = \int_{\boldsymbol{\iota}\in\Lambda} \frac{g(z,\boldsymbol{\iota})}{g(z)}\epsilon(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z),$$

with analogous definitions for $\bar{\eta}(z)$, the welfare-weighted income elasticity, and for $\bar{\varepsilon}^c(z)$, the welfare-weighted compensated elasticity of taxable earnings.[37]

In what follows, it will be convenient to define the household $\boldsymbol{\iota}$ *behavioral wedge*:

$$\xi(z,\boldsymbol{\iota}) \equiv \frac{\partial_z W(z-T(z),z,\boldsymbol{\iota})}{\partial_x W(z-T(z),z,\boldsymbol{\iota})}\frac{1}{1-T'(z)}. \tag{D.4}$$

Analogously to what we have done with elasticities, we define

$$\bar{\xi}(z) = \int_{\boldsymbol{\iota}} \frac{g(z,\boldsymbol{\iota})}{g(z)}\xi(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z),$$

the welfare-weighted average behavioral wedge.

D.1 Optimal tax formula

We have already shown how dissonance changes the direct impact of tax reforms on welfare measured by the planner. Unlike the standard case of single households, the welfare impact of a small reform is not negligible. This happens because the social marginal value of income does not coincide with the private one. Adding the mechanical and behavioral effects on tax revenues along the lines of Saez (2001), we arrive at Proposition 3 at the expression for optimal marginal tax rates.

Proposition 3. *The first-order necessary conditions for the optimal joint tax schedule at the total household income level $z \in \mathbb{R}_+$ can be written as*

$$\begin{aligned}\frac{T'(z)}{1-T'(z)} = \frac{1}{\epsilon^c(z)}\frac{1-\Phi(z)}{z\varphi(z)}&\left\{\int_z^{\bar{z}}\left[1-g(\tilde{z})+\frac{T'(\tilde{z})}{1-T'(\tilde{z})}\eta(\tilde{z})\right]\frac{\varphi(\tilde{z})}{1-\Phi(z)}d\tilde{z}\right.\\ &\left.+\int_z^{\bar{z}} g(\tilde{z})\left\{\left[1-\bar{\xi}(\tilde{z})\right]\bar{\eta}(\tilde{z})-\zeta^{\eta,\xi}(\tilde{z})\right\}\frac{\varphi(\tilde{z})}{1-\Phi(z)}d\tilde{z}\right\}\\ &-\frac{g(z)}{\varepsilon^c(z)}\left\{\left[1-\bar{\xi}(z)\right]\bar{\varepsilon}^c(z)-\zeta^{\varepsilon,\xi}(z)\right\},\end{aligned} \tag{D.5}$$

where $\Phi(z)$ is the empirical income distribution induced by the optimal tax system, $\varphi(z)$ is its density, $g(z)$ is the marginal social value of income in the hands of households with income z, $\bar{\xi}(z)$ is the behavioral wedge between the social values of income and its market price, $\bar{\varepsilon}^c(z)$ and $\bar{\eta}^c(z)$ are welfare-adjusted average elasticities for households choosing z, and $\zeta^{\varepsilon,\xi}(z)$ and $\zeta^{\eta,\xi}(z)$ are the welfare-adjusted covariances between these elasticities and the behavioral wedges for all households choosing z.

The first thing to note about (D.5) is that if $\bar{\xi}(z) = 1$, this is simply the Piketty-Saez intuitive expression for optimal taxes. The second important thing is that two

[37] Of course, if for all z, $\varepsilon(z,\boldsymbol{\iota})$ is the same for all $\boldsymbol{\iota}$ (or if the co-variance between $\partial W/\partial x$ and ϵ over the distribution of $\boldsymbol{\iota}$ is zero), then the two concepts coincide.

types of aggregation are used to characterize optimal taxes: the regular empirical frequency-weighted elasticities ($\varepsilon(z), \varepsilon^c(z)$, and $\eta(z)$) and the welfare-weighted elasticities ($\bar{\varepsilon}(z), \bar{\varepsilon}^c(z)$, and $\bar{\eta}(z)$).

As pointed out by Saez (2001), using a perturbation approach allows us to write optimal tax formulae even when types are multidimensional, which is precisely our case here. However, with multidimensional heterogeneity, households of different types bunch at the same z, and we need to aggregate behavioral responses across these agents.[38] This is already in Saez (2001), as we can see if we assume $\bar{\xi}(z) = 1$. The novelty is that, whereas elasticities are only important in Saez (2001) because of their behavioral impact on revenues, they directly affect utility as perceived by the planner. In this case, averages must be welfare adjusted analogously to risk adjustments in asset pricing formulae.

Dissonance means that behavioral responses have first-order consequences on welfare, which vary across agents with the same earnings. The overall impact, which depends on the elasticities and the wedges, is weighted not by the relative empirical frequency of couples choosing z but by the welfare-adjusted frequencies. All elasticities pertaining to dissonance have a bar to denote this adjustment. The covariance terms that appear in the last two lines of (D.5) are also calculated under this welfare-adjusted measure.[39]

As it turns out, ridding the expression of these aggregation issues leads to a considerably more straightforward extension of optimal tax formulae. Toward this end, we assume that all elasticities are identical for agents choosing the same z. This will lead us to an ABC+D formula (Proposition 2) that is derived from (D.6) below.

Corollary 1. *Assume that for all z, $\varepsilon(z, \boldsymbol{\iota}) = \varepsilon(z)$, $\eta(z, \boldsymbol{\iota}) = \eta(z)$ and $\varepsilon^c(z, \boldsymbol{\iota}) = \varepsilon^c(z)$ for all $\boldsymbol{\iota}$ choosing z. Then, the first-order condition for the optimal joint tax schedule for couples at income level z in Proposition 3 can be rewritten as*

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\epsilon^c(z)} \frac{1-\Phi(z)}{z\varphi(z)} \left\{ \int_z^{\bar{z}} \left[1 - g(\tilde{z}) + \frac{T'(\tilde{z})}{1-T'(\tilde{z})}\eta(\tilde{z})\right] \frac{\varphi(\tilde{z})}{1-\Phi(z)} d\tilde{z} \right.$$
$$\left. + \int_z^{\bar{z}} g(\tilde{z}) \left[1 - \bar{\xi}(\tilde{z})\right] \eta(\tilde{z}) \frac{\varphi(\tilde{z})}{1-\Phi(z)} d\tilde{z} \right\} - \bar{g}(z) \left[1 - \bar{\xi}(z)\right]. \quad \text{(D.6)}$$

Because we have assumed that elasticities are identical for all households choos-

[38]According to Saez (2001) "The direct proof using elasticities shows that it is not necessary to introduce a uni-dimensional exogenous skill distribution to obtain formula (14) [our formula (D.5)]. Therefore, formula (14) might, in principle, be valid for any heterogeneous population as long as (...) are considered as average elasticities at income level z."

[39]Covariance terms also appear in the optimal tax formulae for behavioral agents derived by Gerritsen (2016). Gerritsen's representation slightly differs from ours since he does not work with a change in the measure, considering extra co-variance terms in the empirical measure.

ing the same z, we can drop the bar from almost all statistics and eliminate the covariance terms in (D.5). The only term for which we still make a distinction between the empirical distribution and the welfare-adjusted distribution concerns the behavioral wedge $\bar{\xi}(z)$.

Note how (D.6) identifies a new sufficient statistic that should be considered when evaluating optimal income taxes. It highlights that the planner may want to impose additional distortions to influence decisions taken within the household.

Equation (D.6) defines $T'(z)/(1-T'(z))$ implicitly. We can find an explicit solution by solving this integral equation. This is the content of Proposition 2 in the main text.

Relationship between the optimal taxation formulae Optimal tax formulae (13) could be directly derived from (D.5). First, note that under identical $\ln$ preferences, the marginal social value of resources is equal for all households earning the same z, leading us to drop all of the covariance terms and equalize the empirical and welfare-adjusted moments: the relevant expression for optimal taxes takes the explicit form (17). Second, separability greatly simplifies the $C(z)$ term in (17) once one notes that, under separability, $\eta(z)/\varepsilon^c(z) = d\ln u'(z-T(z))/dz$. A somewhat tedious algebra is needed to show that the dissonance term, $D(z)$, becomes the term in the second line of (13). As in Saez (2001), it is comforting to check that the same formulae, albeit written in terms of primitives only, are reached using a mechanism design approach.

Proof of Proposition 3. The Gateaux derivative of the earning supply functional is defined by

$$dz_{\boldsymbol{\iota}}(T;H) \equiv \lim_{\mu\to 0} \frac{z_{\boldsymbol{\iota}}(T+\mu H) - z_{\boldsymbol{\iota}}(T)}{\mu} = \left.\frac{\partial z_{\boldsymbol{\iota}}}{\partial \mu}\right|_{\mu=0}. \tag{D.7}$$

Increase the marginal tax rate by $d\tau$ in a small interval $(z', z'+dz')$. Let $\Phi(z)$ denote the distribution of income induced by the candidate optimal tax schedule, $T(\cdot)$ (and the distribution $F(\cdot)$ of types $\boldsymbol{\iota}$). Let us then consider the overall impact of such a reform.

Impact on government revenues For agents with taxable income in the interval $(z', z'+dz')$, there is a mechanical effect $M_1 = d\tau dz'$. But there is also a behavioral effect, which is given by

$$B_1(z',\boldsymbol{\iota}) = T'(z') \left.\frac{dz}{d\tau}\right|_u d\tau = T'(z')\epsilon^c(z',\boldsymbol{\iota}) \frac{z'}{1-T'(z')} d\tau.$$

Let $\bar{\varphi}(\boldsymbol{\iota}|z')$ be the conditional distribution on z' of $\boldsymbol{\iota}$. Then, the average response at income z' is

$$B_1(z') = \frac{z'T'(z')}{1 - T'(z')} \int \epsilon^c(z', \boldsymbol{\iota}) d\bar{\varphi}(\boldsymbol{\iota}|z') d\tau = \frac{z'T'(z')}{1 - T'(z')} \epsilon^c(z') d\tau,$$

for $\epsilon^c(z')$ as defined in Section D.

This affects agents in the interval $(z', z' + dz')$, which results in an overall effect

$$\epsilon^c(z') \frac{T'(z')}{1 - T'(z')} z' \varphi(z') dz' d\tau,$$

where φ is the density of z induced by the tax system.

For agents with taxable income in $[z', \bar{z}]$, we have again a mechanical effect $d\tau dz'$ which affects all agents for an overall increase in revenues given by

$$M_2 = [1 - \Phi(z')] \, d\tau dz'.$$

For all agents in this interval, there is also a behavioral response due to the income effect

$$B_2(z, \boldsymbol{\iota}) = T'(z) \frac{dz}{dI} d\tau dz' = \frac{T'(z)}{1 - T'(z)} \eta(z, \boldsymbol{\iota}) d\tau dz',$$

which can be aggregated across all households earning z through

$$B_2(z) = \frac{T'(z)}{1 - T'(z)} \int \eta(z, \boldsymbol{\iota}) d\bar{\varphi}(\boldsymbol{\iota}|z) d\tau dz' = \frac{T'(z)}{1 - T'(z)} \eta(z) d\tau dz'.$$

Finally, we can use the expression above to find the overall behavior effect on tax revenues for agents in this range

$$B_2(z, \boldsymbol{\iota}) = d\tau dz' \int_{z'}^{\bar{z}} \frac{T'(z)}{1 - T'(z)} \eta(z) \varphi(z) dz.$$

This is exactly as in Saez (2001) with the proviso that we are using a different definition for the elasticities we are aggregating.

Welfare impacts As for the welfare effects, again, let us start with households with taxable income in the interval $(z', z' + dz')$. First note that for this group, the tax reform has no first-order mechanical effect on utility, V. The relevant 'decision' elasticities are, therefore, Hicksian.

The impact on welfare is however measured according to the planner's valuation, i.e., the experience' utility W. Hence,

$$dW_1(z',\boldsymbol{\iota}) = \frac{\partial}{\partial x}W(z'-T(z'),z',\boldsymbol{\iota})\left[1-T'(z')\right]\left.\frac{dz}{d\tau}\right|_u d\tau + \frac{\partial}{\partial z}W(z'-T(z'),z',\boldsymbol{\iota})\left.\frac{dz}{d\tau}\right|_u d\tau,$$

which simplifies to

$$dW_1(z',\boldsymbol{\iota}) = \frac{\partial}{\partial x}W(z'-T(z'),z',\boldsymbol{\iota})\left[1-T'(z')+\frac{\frac{\partial}{\partial z}W(z'-T(z'),z',\boldsymbol{\iota})}{\frac{\partial}{\partial x}W(z'-T(z'),z',\boldsymbol{\iota})}\right]\left.\frac{dz}{d\tau}\right|_u d\tau,$$

or

$$dW_1(z',\boldsymbol{\iota}) = \frac{\partial}{\partial x}W(z'-T(z'),z',\boldsymbol{\iota})\left[1-\xi(z',\boldsymbol{\iota})\right]\epsilon^c(z',\boldsymbol{\iota})z'd\tau,$$

where for all z,

$$\xi(z,\boldsymbol{\iota}) \equiv \frac{1}{1-T'(z)}\frac{\frac{\partial}{\partial z}W(z-T(z),z,\boldsymbol{\iota})}{\frac{\partial}{\partial x}W(z-T(z),z,\boldsymbol{\iota})}.$$

Let $\bar{\varphi}(\boldsymbol{\iota}|z')$ the conditional on z' distribution of $\boldsymbol{\iota}$. The overall welfare effect at z' is

$$W_1 = dz'd\tau z'\int\left[g(z',\boldsymbol{\iota})\left[1-\xi(z',\boldsymbol{\iota})\right]\epsilon^c(z',\boldsymbol{\iota})\right]d\bar{\varphi}(\boldsymbol{\iota}|z'),$$

for $g(z',\boldsymbol{\iota})$ the marginal social value of income in the hands of a $\boldsymbol{\iota}$ household earning z'.

To simplify this expression, let $\tilde{\varphi}(\cdot|z)$ be a measure equivalent to $\bar{\varphi}(\cdot|z)$, ($\tilde{\varphi}(\cdot|z) \equiv \bar{\varphi}(\cdot|z)$) whose Radon-Nikodym derivative with respect to $\bar{\varphi}(\cdot|z')$ is the function $g(z,\cdot)/\bar{g}(z)$, where $\bar{g}(z) = \int g(z,\boldsymbol{\iota})d\bar{\varphi}(\boldsymbol{\iota}|z)$. This measure allows us to re-express the definitions for $\bar{\xi}(z)$ and $\bar{\varepsilon}^c(z)$ from Section D, as $\bar{\xi}(z) = \int \xi(z,\boldsymbol{\iota})d\tilde{\varphi}(\boldsymbol{\iota}|z) = \tilde{\mathbb{E}}[\xi(z,\boldsymbol{\iota})]$, and $\bar{\varepsilon}^c(z) = \tilde{\mathbb{E}}[\varepsilon^c(z,\boldsymbol{\iota})]$. It also allows us to define a covariance term

$$\zeta^{\varepsilon,\xi}(z) = c\tilde{o}v\Big(\varepsilon^c(z,\boldsymbol{\iota}),\xi(z,\boldsymbol{\iota})\Big) = \tilde{\mathbb{E}}[\varepsilon^c(z,\boldsymbol{\iota})\xi(z,\boldsymbol{\iota})] - \bar{\xi}(z)\bar{\varepsilon}^c(z).$$

Using these definitions we write

$$W_1 = dz'd\tau z'\bar{g}(z')\left[\left[1-\bar{\xi}(z')\right]\bar{\epsilon}^c(z') - \zeta^{\varepsilon,\xi}(z')\right].$$

For households with taxable income $z \geq z'$,

$$\begin{aligned} dW_2(z,\boldsymbol{\iota}) = &-\frac{\partial}{\partial x}W(z-T(z),z,\boldsymbol{\iota})dz'd\tau + \\ &\frac{\partial}{\partial x}W(z-T(z),z,\boldsymbol{\iota})\left[1-T'(z)\right]\frac{dz}{dI}dz'd\tau + \frac{\partial}{\partial z}W(z-T(z),z,\boldsymbol{\iota})\frac{dz}{dI}dz'd\tau \end{aligned}$$

leading to

$$dW_2(z,\boldsymbol{\iota}) = -\frac{\partial}{\partial x}W(z-T(z),z,\boldsymbol{\iota})\left\{1+\left[1-T'(z)+\frac{\frac{\partial}{\partial z}W(z-T(z),z,\boldsymbol{\iota})}{\frac{\partial}{\partial x}W(z-T(z),z,\boldsymbol{\iota})}\right]\frac{dz}{dI}\right\}dz'd\tau$$

and finally

$$dW_2(z,\boldsymbol{\iota}) = -g(z,\boldsymbol{\iota})\left\{1+\left[1-\xi(z,\boldsymbol{\iota})\right]\eta(z,\boldsymbol{\iota})\right\}dz'd\tau.$$

To aggregate across all household earnings at the same z, we integrate over $\boldsymbol{\iota}$

$$d\bar{W}_2(z) = dz'd\tau\int\Big[-g(z,\boldsymbol{\iota})\left\{1+\left[1-\xi(z,\boldsymbol{\iota})\right]\eta(z,\boldsymbol{\iota})\right\}\Big]d\bar{\varphi}(\boldsymbol{\iota}|z').$$

Analogously, to what we have done for dW_1 we can further simplify this expression to

$$d\bar{W}_2(z) = d\tau dz'\bar{g}(z)\Big[\left[1-\bar{\xi}(z)\right]\bar{\eta}^c(z)-\zeta^{\eta,\xi}(z)\Big],$$

where the covariance term $\zeta^{\eta,\xi}(z)$ is also under the equivalent measure, $\tilde{\varphi}(\cdot|z)$.

The overall effect on this household welfare is

$$W_2 = d\tau dz'\int_{z'}^{\bar{z}}\bar{g}(z)\left\{1+\left[1-\bar{\xi}(z)\right]\bar{\eta}(z)-\zeta^{\eta,\xi}(z)\right\}\varphi(z)dz.$$

At the optimum, all these effects must cancel out and we get

$$M_1+M_2+B_1+B_2+W_1+W_2=0.$$

Substituting the expressions for each one of these terms we get

$$\begin{aligned}&\frac{T'(z')}{1-T'(z')}\epsilon^c(z')z'\varphi(z')-g(z')\Big[\left[1-\xi(z')\right]\bar{\varepsilon}^c(z')-\zeta^{\varepsilon,\xi}(z')\Big]z'\varphi(z')+\left[1-\Phi(z')\right]\\&+\int_{z'}^{\bar{z}}\frac{T'(z)}{1-T'(z)}\eta(z)\varphi(z)dz+\int_{z'}^{\bar{z}}g(z)\left\{1+\left[1-\xi(z)\right]\bar{\eta}(z)-\zeta^{\eta,\xi}(z)\right\}\varphi(z)dz=0.\end{aligned}$$

Reorganizing the terms,

$$\begin{aligned}\frac{T'(z')}{1-T'(z')}\epsilon^c(z')z'\varphi(z') &= \int_{z'}^{\bar{z}}\left[1-g(z)+\frac{T'(z)}{1-T'(z)}\eta(z)\right]\varphi(z)dz\\&+\int_{z'}^{\bar{z}}g(z)\left\{\left[1-\xi(z)\right]\bar{\eta}(z)-\zeta^{\eta,\xi}(z)\right\}\varphi(z)dz\\&+g(z')\Big[\left[1-\xi(z')\right]\bar{\varepsilon}^c(z')-\zeta^{\varepsilon,\xi}(z')\Big]z'\varphi(z')\end{aligned}$$

or

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\epsilon^c(z)}\frac{1-\Phi(z)}{z\varphi(z)}\left\{\int_z^{\bar{z}}\left[1-g(\tilde{z})+\frac{T'(\tilde{z})}{1-T'(\tilde{z})}\eta(\tilde{z})\right]\frac{\varphi(\tilde{z})}{1-\Phi(z)}dz\right.$$
$$\left.+\int_{z'}^{\bar{z}} g(\tilde{z})\left\{[1-\xi(\tilde{z})]\,\bar{\eta}(\tilde{z})-\zeta^{\eta,\xi}(\tilde{z})\right\}\frac{\varphi(\tilde{z})}{1-\Phi(z)}dz\right\}$$
$$-\frac{g(z)}{\varepsilon^c(z)}\left\{[1-\xi(z)]\bar{\varepsilon}^c(z)-\zeta^{\varepsilon,\xi}(z)\right\}. \quad \text{(D.8)}$$

□

Proof of Proposition 1. We prove Proposition 1 as a particular case of Proposition 3. Under the assumptions of Proposition 1, $\xi(z) = \bar{\xi}(z)$, $\varepsilon^c(z) = \bar{\varepsilon}^c(z)$ and $\eta(z) = \bar{\eta}c(z)$, for all z. Moreover, $\zeta^{\varepsilon,\xi}(z) = 0$ and $\zeta^{\eta,\xi}(\tilde{z}) = 0$, for all z. As a consequence, expression (D.5) collapses to (D.6).

If we define

$$\kappa(z) := \frac{T'(z)}{1-T'(z)},$$

$$A(z)^{-1} := \frac{1}{\epsilon^c(z)}\frac{1-\Phi(z)}{z\varphi(z)},$$

$$B(z) := [1-g(z)+g(z)[1-\xi(z)]\eta(z)]\,\frac{\varphi(\tilde{z})}{1-\Phi(z)},$$

$$C(z) := \eta(z)\frac{\varphi(\tilde{z})}{1-\Phi(z)}$$

and

$$D(z) = g(z)[1-\xi(z)]A(z),$$

then (D.8) can be written as

$$A(z)\kappa(z) = \int_z^{\bar{z}}\left[B(\tilde{z})+C(\tilde{z})\kappa(\tilde{z})\right]d\tilde{z} - D(z)$$

and solved for

$$\kappa(z) = -\frac{1}{A(z)}\int_z^{\bar{z}}\left[B(\tilde{z})+\frac{D(\tilde{z})C(\tilde{z})}{A(\tilde{z})}-D(\bar{z})\right]\exp\left\{\int_z^{\tilde{z}}\frac{C(\tilde{\tilde{z}})}{A(\tilde{\tilde{z}})}d\tilde{\tilde{z}}\right\}d\tilde{z}+\frac{D(z)}{A(z)}$$

or

$$\frac{T'(z)}{1-T'(z)} = \frac{1}{\varepsilon^c(z)}\frac{1-\Phi(z)}{\varphi(z)z}\int_z^{\bar{z}}\{g(\tilde{z})-1\}\exp\left\{\int_{\tilde{z}}^{z}\frac{\eta(\tilde{\tilde{z}})}{\varepsilon^c(\tilde{\tilde{z}})}\frac{d\tilde{\tilde{z}}}{\tilde{\tilde{z}}}d\tilde{\tilde{z}}\right\}\frac{\varphi(\tilde{z})}{1-\Phi(z)}d\tilde{z}$$
$$-g(z)\{1-\xi(z)\}. \quad \text{(D.9)}$$

□

E Gender-based Taxes

We consider the impact of a reform that increases w_a, i.e., $\hat{w}_a = w_a(1-t)$ for $t < 0$ while adjusting $T(\cdot)$ to hold z fixed.

The household optimal choice of z_a and z_b to attain earnings z is

$$\min_{z_a} \left\{ \alpha \left(\frac{z_a}{w_a} \right)^{1+\gamma} + (1-\alpha) \left(\frac{z - z_a}{w_b} \right)^{1+\gamma} \right\}.$$

The first-order condition is

$$\alpha \frac{z_a^\gamma}{w_a^{1+\gamma}} - (1-\alpha) \frac{(z-z_a)^\gamma}{w_b^{1+\gamma}} = 0.$$

Now consider a reform that increases w_a but changes $T(\cdot)$ in such a way as to keep z fixed. In this case,

$$\left[\gamma\alpha \frac{z_a^{\gamma-1}}{w_a^{1+\gamma}} + \gamma(1-\alpha) \frac{(z-z_a)^{\gamma-1}}{w_b^{1+\gamma}} \right] dz_a - \alpha(1+\gamma) \frac{z_a^\gamma}{w_a^{2+\gamma}} dw_a = 0$$

or

$$\left[\frac{z_a^\gamma}{w_a^{1+\gamma}} + \frac{1-\alpha}{\alpha} \frac{z_b^\gamma}{w_b^{1+\gamma}} \frac{z_a}{z_b} \right] \frac{dz_a}{z_a} - \frac{1+\gamma}{\gamma} \frac{z_a^\gamma}{w_a^{1+\gamma}} \frac{dw_a}{w_a} = 0.$$

Using the first-order condition,

$$\left[\frac{z_a^\gamma}{w_a^{1+\gamma}} + \frac{z_a^\gamma}{w_a^{1+\gamma}} \frac{z_a}{z_b} \right] \frac{dz_a}{z_a} - \frac{1+\gamma}{\gamma} \frac{z_a^\gamma}{w_a^{1+\gamma}} \frac{dw_a}{w_a} = 0,$$

which can be simplified to

$$\left[1 + \frac{z_a}{z_b} \right] \frac{dz_a}{z_a} - \frac{1+\gamma}{\gamma} \frac{dw_a}{w_a} = 0.$$

Hence,

$$\frac{dz_a}{z_a} = \frac{z_b}{z_a + z_b} \frac{1+\gamma}{\gamma} \frac{dw_a}{w_a}.$$

An increase in w_a leads to an increase in n_a whenever $k_a < 1/(1+\gamma)$. Because z is held fixed in this exercise, one is necessarily transferring utility to the other spouse. More generally, we have that

$$\left.\frac{dV_a}{dw_a}\right|_z = -n_a^\gamma \left.\frac{dn_a}{dw_a}\right|_z = -\frac{n_a^\gamma}{w_a} \left[\frac{z_b}{z_a + z_b} \frac{1+\gamma}{\gamma} - 1 \right] \frac{z_a}{w_a}$$

and

$$\left.\frac{dV_b}{dw_a}\right|_z = -n_b^\gamma \left.\frac{dn_b}{dw_a}\right|_z = \frac{n_b^\gamma}{w_b}\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma}\frac{z_a}{w_a}.$$

In this case,

$$\begin{aligned}\left.\frac{dV_a}{dw_a}\right|_z - \left.\frac{dV_b}{dw_a}\right|_z &= -\frac{n_a^\gamma}{w_a}\left\{\left[\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma}-1\right]-\frac{n_b^\gamma}{w_b}\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma}\right\}n_a \\ &= \left\{-\left[1+\frac{\alpha}{1-\alpha}\right]\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma}+1\right\}\frac{n_a^{\gamma+1}}{w_a} \\ &= \left\{-\frac{1}{1-\alpha}\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma}+1\right\}\frac{n_a^{\gamma+1}}{w_a},\end{aligned}$$

which implies that

$$\operatorname{sgn}\left\{\left.\frac{dV_a}{dw_a}\right|_z - \left.\frac{dV_b}{dw_a}\right|_z\right\} = \operatorname{sgn}\left\{1-\frac{1}{1-\alpha}\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma}\right\}.$$

Note that

$$1-\frac{1}{1-\alpha}\frac{z_b}{z_a+z_b}\frac{1+\gamma}{\gamma} \leq 0 \Leftrightarrow \frac{z_a+z_b}{z_b} \leq \frac{1}{1-\alpha}\frac{1+\gamma}{\gamma}.$$

If b is the primary earner, then, $\frac{z_a+z_b}{z_b} \leq 2$, and a sufficient condition is

$$2 \leq \frac{1}{1-\alpha}\frac{1+\gamma}{\gamma}.$$